AD-775 632

COMPUTER PROGRAMS FOR THE ANALYSIS OF SPACECRAFT MAGNETISM

Milton H. Lackey

Naval Ordnance Laboratory
White Oak, Maryland

28 September 1973

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

AD 775 632

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

U. S. Naval Ordnance Laboratory
White Oak, Silver Spring, Maryland

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

COMPUTER PROGRAMS FOR THE ANALYSIS OF SPACECRAFT MAGNETISM

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Magnetic Data Analysis Report

5. AUTHOR(S) (First name, middle initial, last name)

Milton H. Lackey

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
28 September 1973	167	5

8a. CONTRACT OR GRANT NO.

b. PROJECT NO. NOL-786-G01-53

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

NOLTR 73-191

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Naval Research Laboratory
Washington, D. C. 20390

13. ABSTRACT

This report describes five computer programs that are being used at the Naval Ordnance Laboratory for analyzing satellite magnetism. The programs contain numerical analysis algorithms for the spherical harmonic analysis of the magnetic field emanating from a satellite. The analysis is directed at determining the components of the magnetism which correspond to the dipole moment, quadrupole moment, etc. The first three programs were devised to analyze data from magnetic field measurements around the satellite. The fourth program was devised to generate simulated measurement data for a specified system of multipole magnets. The last program is a combination of the data generation and the data analysis programs. Sample problems are included in the discussion to illustrate the techniques of using the programs with a CDC 6400 Computer including the INTERCOM time-sharing system. A brief description is also included of data acquisition techniques and of principal subprograms.

i.

165

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

UNCLASSIFIED

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Associated Legendre Polynomial						
Computer Programs						
Spacecraft						
Numerical Analysis						
Satellite Magnetism						

NOLTR 73-191 28 September 1973

Computer Programs for the Analysis of Spacecraft Magnetism

During the past several years techniques have been developed for measuring and analyzing the magnetism of spacecraft. The techniques are directed, first, at estimating the spacecraft's magnetic dipole moment and, second, at compensating the dipole moment to allow the spacecraft to maintain a stable orientation while in orbit. This procedure requires accurate measurements of the magnetic field emanating from the spacecraft. The Naval Ordnance Laboratory is currently involved in the development of a facility to conduct sophisticated magnetic tests of spacecraft. The facility will contain instrumentation which will automatically record and analyze the test data. This report has been published to document the numerical techniques to be used in the analysis of the data. Techniques are also described which can be used to predict the accuracy of different types of measurement and analysis techniques.

The development and testing of the computer programs required a considerable amount of effort. Mr. H. W. Korab contributed much to this effort, including the development of the BASIC version of the analysis procedure. He also assisted in the development of the illustrations included in this report.

This work was funded in part by the Spacecraft Technology Center of the Naval Research Laboratory under task number NOL-786-GO1-53.

ROBERT WILLIAMSON II
Captain, USN
Commander

R B Knowles

R. B. KNOWLES
By direction

CONTENTS

CONTENTS (CONT)

ILLUSTRATIONS

ILLUSTRATIONS (CONT)

TABLES

REFERENCES

(a) NOLTR 69-60, The Reaction of a Rigid Body to a Uniform Force Field, 17 Mar 1969, by M. H. Lackey

(b) S. Chapman & J. Bartells, Geomagnetism (Oxford Press, London, 1940), Vol. II

(c) NASA TM X-63765, A New Method for Determining the Magnetic Dipole Moment of a Spacecraft from Near-field Data, August 1969, by W. L. Eichhorn

(d) NOLTR 73-128, Proposed Modernization of the Spherical Field Coil Facility, Building 203, 11 June 1973, by M. H. Lackey

(e) NOL ltr 533:MHL:gs 3900 Ser: 5154 of 5 Sep 1973 to NRL, Subj: Magnetic Test Facility for Satellites, instrumentation for

Chapter 1

INTRODUCTION

1. A major effort has been applied to the development of computer programs and subprograms to assist in the analysis of satellite magnetism. The analysis is part of a procedure directed at determining and compensating the magnetic dipole moment of a satellite. The procedure involves four steps. The first step is to measure the normal component of the satellite's magnetic field on the surface of a sphere enclosing the satellite. The second step involves the spherical harmonic analysis of the measured data to determine the magnitude and the direction of the satellite's dipole moment. The third step is to attach an opposing dipole to the satellite with equal magnitude and opposite direction. The final step is to repeat the measurement and analysis procedures to verify the compensation.

2. The five programs described in this report have been devised to perform a variety of data analysis tasks. One of the programs is coded in the BASIC computer language. The other four are coded in FORTRAN IV. The first three programs are used to analyze data representing the normal component of the magnetic field on the surface of a sphere enclosing the satellite. The fourth program is used to generate simulated measurement data for a specified system of multipole magnets. The program also allows the simulation of measurement errors. The last program is a combination of the data generation and the analysis programs. The programs have been designed to allow a variety of options including:

a. Reading the input data from paper tape or from a data file

b. Using either an algebraic or geometric integrating scheme to perform the analysis

c. Printing the data after preliminary data processing

d. Interpolating and plotting the processed data curves.

3. The discussion begins with a brief description of data acquisition techniques. This is followed by a description of the five programs including sample problems which demonstrate the use of the programs. Finally, a description is given of some of the primary subprograms. These are listed separately since they can be used as building blocks for other programs. The discussion is supplemented by several illustrations, many of which are the actual computer outputs from the execution of the computer programs. The majority of the data curves are output from the CALCOMP 570 digital incremental plotter. A description of the data units for the programs has also been included in paragraph 33. Appendix A contains a glossary of symbols and terms used in the report.

Chapter 2

DATA ACQUISITION TECHNIQUES

4. A brief description of spherical coordinates and the techniques used in the data acquisition will assist in the description of the parameters used in the data analysis and the computer programs. Figure 1 illustrates the relationship between rectilinear and spherical coordinates. An arbitrary point $\bar{R}$ (or vector) in space can be defined in terms of the spherical coordinates of R, θ, and φ representing the radial distance, the colatitude and the (easterly) longitude respectively. (θ is sometimes called the polar angle, and φ is sometimes called the dihedral angle.) Appendix A gives the definitions of these coordinates in terms of rectilinear coordinates.

5. Figure 2 shows a simplified diagram of the test setup for conducting the measurements of a satellite's magnetism. The degrees of freedom illustrated in the figure are defined to correspond to the spherical angles. The rotation axis, provided by a horizontal turntable, allows a variation in colatitude. The tilt axis, provided by a gimballing fixture attached to the satellite, allows a variation in longitude. Analog curves are made of the sensor reading versus the colatitude for a fixed set of positions of longitude. The curves are recorded while the satellite is in a zero magnetic field environment. Notice that the sensor in Figure 2 is radially aligned. This setup generates analog curves representing the normal component of the satellite magnetism along great circles of longitude as shown in Figure 3.

6. Let the parameter NO represent the number of curves of data to be recorded. These curves correspond to measurements along great circles spaced (180/NO) degrees apart. Figure 3 shows an example when NO = 6. There will be six analog curves of data corresponding to the six great circles. Figure 4 shows the initial positions for each of the curves.

7. As a sample problem consider the system of dipoles illustrated in Figure 5. The first curve begins with the +z-axis directed at the magnetic sensor and the +y-axis pointed up. The turntable is rotated clockwise. Prior to beginning each succeeding curve, the satellite is tilted (180/N0 = 30) degrees around the z-axis. Figure 6 shows a typical set of six data curves for the sample problem. Each curve is marked with 25 data points. The parameter N1 is used to represent the number of data points per curve (N1 = 25 in the example). The points are spaced (360/(N1-1)) = 15 degrees apart. The value at each measurement point is automatically recorded via analog-to-digital (A/D) conversion equipment in the satellite measurements.

8. Notice that the curves in Figure 6 do not all begin with the same value, although, theoretically they should. This is typical of the type of measurements that are made on satellites. The curves represent relative measurements instead of absolute measurements. It would be possible to insure that the curves all begin with the same value, but it would not necessarily be the correct value. Nevertheless, the data analysis is independent of the starting value of each curve. Therefore, no concern is given to this problem in the acquisition of the data.

9. Reference (a) lists several considerations in the determination of the parameters for the data acquisition including N0, N1, and the radius R1 of the measurement sphere. The considerations include:

a. The smoothness of the data curves

b. The analysis method

c. The accuracy of the measuring apparatus

d. The round-off errors of the computing machine.

The parameters are not all independent. They must be determined by making certain compromises. For instance, the data curves can be smoothed by increasing R1 (i.e., by increasing the minimum distance between the surface of the measurement sphere and the satellite), but this will decrease the relative accuracy of the data. Also, the total number of data points N0•N1 may be increased, but computing machine errors will become more significant. An increase in the number of data points will also increase the computing time and data storage.

10. Experience has shown that the dipole moment for most problems can be approximated accurately enough if the following conditions hold:

a. Let the minimum distance between the surface of the measurement sphere and the satellite be at least half the maximum diameter of the satellite to insure sufficient smoothness of the data curves. For example, the radius of the measurement sphere for a body with maximum diameter of 50 inches, and centered at the origin, should be at least 50 inches.

b. Let NO be ≥ 8 and N1 be ≥ 16 to insure that the errors from the numerical approximation are smaller than the measurement errors. (This condition can be relaxed in cases where the satellite magnetism gives simple sine-cosine curves. In the past, NO = 4 and N1 = 25 has been sufficient in many cases.)

c. Use the simplest analysis procedure (i.e., the geometric integrating scheme) for the preliminary analysis since it is relatively accurate and easy to use.

Chapter 3

MAIN PROGRAMS

PROGRAMS SA1024 AND SA2024

Introduction

11. The programs labeled SA1024 and SA2024 (Appendices B and C) are BASIC and FORTRAN IV versions, respectively, of the basic one-term data analysis procedure. The procedure is simple because only the dipole moment term is computed for the satellite magnetism and only the approximate method of numerical integration is used. The programs print the measurement data and the computed dipole moment in rectilinear and spherical coordinates. They also print data which assists in calibrating and aligning the compensating dipoles. The major difference between the two versions is the manner of entering the data. The BASIC version is set up to take the data from DATA statements. The FORTRAN IV version accepts data from paper tape or from a data file.

Input Data

12. The data is entered into the programs in the order of the following definitions:

a. First line of data or data card - format: (3I5)

NO - The number of curves of data (NO is even and ≤ 16.)

N1 - The number of equally spaced data points per curve from 0 thru 360 degrees colatitude (N1 is odd and ≤ 33. The first and last data points for each curve correspond to measurements at 0 degrees colatitude.)

IR - The parameter that determines whether or not to read the data from a data file. IR ≠ 0 means that the data will be read from the data file DAT024. (This parameter is used only with the FORTRAN IV version.)

b. <u>Second line of data or data card - format: (2F6.1)</u>

R1 - The radius of the measurement sphere in inches

C6 - The value of the calibration signal (C7-C8) in gamma

c. <u>Third line of data or data card - format: (I5)</u>

N2 - The parameter that determines whether or not to print the measured data ($N2 \neq 0$ means that the data will be printed.)

d. <u>Measurement data - format: (9F6.1) (These cards are deleted if $IR \neq 0$.)</u>

C7 - The static measurement with the calibration signal

C8 - The static measurement without the calibration signal

F(I,J) - The measurement data for I = 1,2,···,N1 and J = 1,2,···,NO.

The input data cycle may be repeated by starting with new data NO, N1, and IR again. The program execution is terminated by setting the new value for NO equal to zero.

<u>Output Data</u>

13. The programs SA1024 and SA2024 then perform a numerical integration of the equation

$$\bar{D} = (3 \cdot R1^3/8\pi) \int_0^{\pi} \int_0^{2\pi} f(\theta,\varphi) \sin\theta \, d\theta \, d\varphi \tag{1}$$

for the dipole moment $\bar{D}$. The numerical equation is

$$\bar{D} \approx (3 \cdot R1^3/8\pi) \sum_{J=1}^{NO} \sum_{I=1}^{N1} F(I,J) \cdot \pi(\theta(I),\varphi(J)) \cdot X(I) \tag{2}$$

where

$\bar{D} \equiv (D_1, D_2, D_3)$ is in gauss-centimeters^3

$F(I,J) \equiv f(\theta(I),\varphi(J))$ is in gammas

$$\bar{n}(\theta(I),\varphi(J)) \equiv \begin{bmatrix} \sin(\theta(I))\cdot\cos(\varphi(J)) \\ \sin(\theta(I))\cdot\sin(\varphi(J)) \\ \cos(\theta(I)) \end{bmatrix}$$

$$\theta(I) \equiv 2\pi(I-1)/(N1-1) \text{ is the colatitude (turntable angle)} \quad (3a)$$

$$\varphi(J) \equiv \pi(J-1)/NO \text{ is the longitude (tilt angle)} \quad (3b)$$

$$X(I) \equiv [\sin(\pi/(2\cdot N1-2))]^2/(2\cdot NO) \text{ if } I = 1 \text{ or } N1 \quad (4a)$$

$$\equiv [\sin(\pi/(2\cdot N1-2))]^2/NO \text{ if } I = (N1+1)/2 \quad (4b)$$

$$\equiv |\sin(\theta(I))\cdot\sin(\pi/(N1-1))|/(2\cdot NO) \text{ if } I \neq 1, (N1+1)/2, \text{ or } N1. \quad (4c)$$

14. Next, the dipole moment is transformed into spherical coordinates as

$$D \equiv (D_1^2 + D_2^2 + D_3^2)^{\frac{1}{2}}$$

$$\theta \equiv \tan^{-1}\left[(D_1^2 + D_2^2)^{\frac{1}{2}}/D_3\right]\cdot 180/\pi = \text{colatitude}$$

$$\varphi \equiv \tan^{-1}(D_2/D_1)\cdot 180/\pi = \text{longitude}$$

15. Finally, the program computes the values of two compensating magnets: one in the xy-plane and one along the z-axis. The value of the dipole field at one meter is also given to assist in charging the magnets to the desired values.

Sample Problem for SA2024

16. Appendix D contains the input and output data for SA2024 using the sample problem represented in Figures 5 and 6. The file BN2024, which is used for execution, is the binary version of SA2024. This example was executed in two different ways on the INTERCOM time-sharing system using the CDC 6400 computer. The information typed in at the teletype terminal has been underlined. Data tapes representing the measurement data are generated by the A/D equipment during the measurements. The program automatically reads the data in the proper format; converts the data into gammas using the calibration parameters C6, C7, and C8; and adjusts the data so that the beginning and ending points for all the curves have nearly the same value. The result of this procedure is visible in the data print-out on page 2 of Appendix D.

17. The first method of data input in Appendix D was from the file DAT024. This complete problem required only the first three lines of data. The measurement data was read into the file from an earlier execution of SA2024. (The program execution automatically generates the file DAT024 if the data is read in from tape.) The remaining lines of data initiated the computation of the solution to the same problem except that the measurement data was read in from a data tape.

PROGRAM SA3024

Introduction

18. The program SA3024, listed in Appendix E, allows a more complete analysis to be performed on the data. The program can be used to compute the dipole, quadrupole, and higher order multipole terms. Also included is an optional method of integrating the data and a data plotting option. The optional integrating scheme is labeled "exact" although it is exact only for magnetic data from a finite number of multipole magnets centered at the origin. This scheme is discussed in more detail in paragraphs 25 through 32 and in reference (a).

19. The program uses several special subprograms to perform such tasks as:

a. The computation of spherical harmonic coefficients for magnetic field data

b. The generation of values of associated Legendre polymomials and Schmidt functions

c. The inversion of an n x n matrix

d. The interpolation and plotting of data.

Some of these subprograms will be discussed in more detail in later sections.

Input Data

20. The method for entering the data into SA3024 is very similiar to the method for SA2024 with several additional variables. The data is entered in the following order:

a. First line of data or data card - format: (6I5)

NO - The number of curves of data (NO is even and $\leq$ 16.)

N1 - The number of equally spaced data points per curve from 0 thru 360 degrees colatitude (N1 is odd and ≤ 33. The first and last data points for each curve correspond to measurements at 0 degrees colatitude.)

NH - The highest degree spherical harmonic term to be computed from the data (NH = 1 for dipoles, 2 for quadrupoles, etc.)

IR - The parameter that determines whether or not to read the data from a data file (IR $\neq$ 0 means that the data will be read from the file DAT024.)

IP1 - The parameter that determines whether or not the data is to be interpolated and plotted (IP1 $\neq$ 0 means that the data will be interpolated and plotted.)

IW - The parameter that determines which integrating scheme is to be used (IW = 0 means that the exact, algebraic scheme is to be used.)

b. Second line of data or data card - format: (3F8.4)

R1 - The radius of the measurement sphere in inches

CV - The value of the calibration signal (CS-CZ) in gamma

PY - The scale factor (gammas/inch) for the y-axis if the data is to be plotted (If PY = 0.0 a factor will be computed from the data.)

c. Third line of data or data card - format: (I5)

IP2 - A parameter that determines whether or not to print the measured data (IP2 $\neq$ 0 means that the data will be printed.)

d. Measurement data - format: (9F6.1) (These cards are deleted if IR $\neq$ 0.)

CS - The static measurement with the calibration signal

CZ - The static measurement without the calibration signal

F(I,J) - The measurement data for I = 1,2,···,N1 and J = 1,2,···,NO.

The input data cycle may be repeated by starting with new data NO, N1, ···, etc. The program is terminated by setting the new value for NO equal to zero.

Output Data

21. The program then computes the spherical harmonic coefficients A_n^m and B_n^m based on the equation

$$\begin{Bmatrix} A_n^m \\ B_n^m \end{Bmatrix} = \left[(2n + 1)\cdot R1^{n+2}/(4\pi\cdot(n + 1))\right] \int_0^{\pi}\int_0^{2\pi} f(\theta,\varphi)\cdot P_n^m\,(\cos\,\theta) \begin{Bmatrix} \cos(m\varphi) \\ \sin(m\varphi) \end{Bmatrix}\cdot$$

$$\cdot\sin\,\theta d\theta d\varphi \tag{5}$$

for n = 0,1,···,NH and m = 0,1,···,n. The numerical equivalent of equation (5) is

$$\begin{Bmatrix} A_n^m \\ B_n^m \end{Bmatrix} = \left[(2n + 1)\cdot R1^{n+2}/(4\pi\cdot(n + 1))\right] \sum_{J=1}^{NO} \sum_{I=1}^{N1} F(I,J)\cdot P_n^m(\cos(\theta(I)))\cdot$$

$$\cdot\begin{Bmatrix} \cos(m\cdot\varphi(J)) \\ \sin(m\cdot\varphi(J)) \end{Bmatrix}\cdot Y(I) \tag{6}$$

where

n is the degree of the spherical harmonic term

m is the order of the spherical harmonic term

$P_n^m(\cos(\theta(I)))$ is the Schmidt function of degree n and order m. (These functions are discussed in Appendix K.)

Y(I) is the array of weighting factors for the numerical integration. (One of two methods can be used to determine the values for Y(I) depending on the parameter IW.)

$\theta(I) \equiv 2\pi(I - 1)/(N1 - 1)$ is the colatitude (turntable angle)

$\varphi(J) \equiv \pi(J - 1)/NO$ is the longitude (tilt angle).

Equation (5) gives an expansion for the function $f(\theta,\varphi)$ as

$$f(\theta,\varphi) = \sum_{n=0}^{\infty} (n+1)/(R1^{n+2}) \sum_{m=0}^{n} \left[A_n^m \cos(m\varphi) + B_n^m \sin(m\varphi)\right] P_n^m(\cos(\theta)). \qquad (7)$$

If the expansion is written as $f(\theta,\varphi) = \sum_{n=0}^{\infty} f_n(\theta,\varphi)$ then each $f_n(\theta,\varphi)$ represents the field from a multipole magnet of degree n. (Reference (a) contains more details on the expansion of the function $f(\varphi,\theta)$.)

22. In general, the coefficients A_n^m and B_n^m can be stored as two-dimensional arrays A(n,m) and B(n,m), or, they may be packed into single-dimensional arrays as

$$A(m + 1 + (n^2 + n)/2) \equiv A_n^m \text{ for } n = 0,1,2,\cdots \text{ and } m = 0,1,\cdots,n$$

$$B(m + (n^2 - n)/2) \equiv B_n^m \text{ for } n = 1,2,\cdots \text{ and } m = 1,2,\cdots,n.$$

23. The program SA3024 is set up to conserve storage by computing and storing the coefficients for only one degree term at a time (i.e., for each fixed n) using the equations

$$A(m + 1) \equiv A_n^m \text{ for } m = 0,1,\cdots,n$$

$$B(m) \equiv B_n^m \text{ for } m = 1,2,\cdots n. \qquad (8)$$

It should be noted that the coefficient A_0^0, corresponding to the monopole moment, would be zero if the numerical integration was exact and the data F(I,J) was correct. If the monopole moment is not zero, then the data is corrected prior to being printed and prior to further analysis. Also for n = 1 and Y(I) = X(I) it can be shown that Eqs. (1) (or (2)) and (5) (or (6)) are identical if

$$D_1 = A_1^1$$

$$D_2 = B_1^1$$

$$D_3 = A_1^0. \qquad (9)$$

24. The dipole moment $\bar{D} = (D_1, D_2, D_3)$ is computed and printed out separately in the program in both rectilinear and spherical coordinates. The program is also

set up to compute the quadrupole moment Q_{ij} for i and j = 1, 2, and 3. This is computed and printed separately if NH ≥ 2. The computation is based on the equations

$$Q_{11} = \sqrt{3} \cdot A_2^2 - A_2^0 \tag{10a}$$

$$Q_{22} = -\sqrt{3} \cdot A_2^2 - A_2^0 \tag{10b}$$

$$Q_{33} = 2A_2^0 \qquad (\text{i.e., } Q_{11} + Q_{22} + Q_{33} = 0) \tag{10c}$$

$$Q_{12} = \sqrt{3} \cdot B_2^2 \tag{10d}$$

$$Q_{13} = \sqrt{3} \cdot A_2^1 \tag{10e}$$

$$Q_{23} = \sqrt{3} \cdot B_2^1 \tag{10f}$$

$$Q_{ij} = Q_{ji} \text{ for i and j = 1, 2, and 3.} \tag{10g}$$

The higher degree coefficients for n = 3,4,···,NH are printed only in their spherical form.

Integrating Schemes

25. The program SA3024 is set up to allow a choice between two numerical integration schemes for Eq. (6). These two schemes are based on two different methods for determining the weighting factors $\{Y(I)\}$. The factors actually correspond to the elements of spherical surface area assigned to each data point F(I,J). Therefore, a geometrical description of the two different sets of weighting factors will provide some insight into the two methods of numerical integration. Figure 7 shows an example of the areas assigned to a data point for both integrating methods. The dashed lines are boundaries for areas when the parameter IW ≠ 0. The areas have values $\{X(I)\}$ as defined in Eq. (4). (The weighting factors $\{Y(I)\}$ in Eq. (6) are then set equal to $\{X(I)\}$.) The dashed lines are equally spaced between data points with equal intervals of longitude (= π/NO) and equal intervals of colatitude (= 2π/N1). The weighting factors $\{Y(I) = X(I)\}$ are easier and faster to calculate by computer than the factors when IW = 0. The resulting integration has good numerical stability and gives fairly accurate answers.

26. If the parameter IW = 0, then a more exact integrating scheme is used. The weights $\{Y(I)\}$ are still elements of area on the surface of the unit sphere, and they still consist of equal intervals of longitude. But the intervals of colatitude (dotted lines in Figure 7) are varied to make certain surface integrals exact if the integrand consists of a finite number of spherical harmonic terms. The highest degree term that can be contained in the integrand (or highest degree magnet that can be represent by the data F(I,J)) and still be exact depends on the parameters NO and N1. The relationship of NO and N1 to the degree n and order m of the coefficients A_n^m and B_n^m are

$$\begin{aligned} NO &\geq m + 1 \\ N1 &\geq 4n + 1. \end{aligned} \tag{11}$$

Since $m \leq n$, as seen in Eqs. (5) and (7), the coefficients for a multipole magnet of degree n can be approximated accurately only if

$$\begin{aligned} NO &\geq n + 1 \\ N1 &\geq 4n + 1. \end{aligned} \tag{12}$$

This means that the dipole terms (n = 1) require that NO ≥ 2 and N1 ≥ 5. The following table shows values for several multipole magnets.

TABLE 1 EXAMPLES OF MINIMUM VALUES OF NO AND N1

n	NO≥	N1≥
1 (dipole)	2	5
2 (quadrupole)	3	9
3	4	13
4	5	17
5	6	21
6	7	25
7	8	29
8	9	33

27. It should be noted here that setting NO, N1 = 2, 5 will not generally give the dipole term very accurately unless the higher degree terms are all zero. For example, consider a problem which has only dipole and quadrupole terms, and assume

that only the dipole coefficients are to be computed. Then NO, N1 ≥ 3, 9 is required to insure the accuracy of the dipole computations. In general a problem contains an infinite number of terms. The only cases when the expansion in Eq. (7) contains a finite number of terms are when there are only a pure dipole (of insignificant length), a quadrupole, and/or, a finite number of other multipole magnets which are centered at the origin. An offset dipole or a dipole of considerable length (or any offset multipole magnet of degree n) requires an infinite number of terms for representation by Eq. (7). In general, it is best to select NO and N1 as large as possible with

$$NO = (N1 - 1)/2. \tag{13}$$

28. For IW = 0, the weights Y(I) are composed of two factors D(I) and C, i.e.,

$$Y(I) = D(I)\cdot C \tag{14}$$

for I = 1,2,···,N1 and J = 1,2,···,NO (see reference (a)). The constant C represents the equally spaced intervals of longitude with value

$$C = \pi/NO. \tag{15}$$

The factors D(I) are determined by solving a set of simultaneous linear equations. The equations can be set up in a number of ways since the factors are symmetric with respect to the values of colatitude of $\theta = \pi/2$ and $\theta = \pi$. The method used in SA3024 is to set up and solve the equations for factors representing the intervals between $\theta = 0$ and $\pi/2$, and then to use symmetry to determine the other weights. This procedure involves two different cases based on the odd integer N1. Figure 8 shows the intervals of colatitude for two examples; one when (N1 + 1) is a multiple of 4 and one when it is not. The examples give rise to two different sets of equations for the factors D(I) as follows:

a. <u>Equations when (N1 + 1) is a multiple of 4</u>

Let

$$\theta(I) \equiv 2\pi(I - 1)/(N1 - 1) \tag{3a}$$

$$N3 \equiv \text{largest integer} \le (N1 + 3)/4 \tag{16}$$

Then

$$\sum_{J=1}^{N3} D(J) = 1 \tag{17a}$$

$$\sum_{J=1}^{N3} (\cos\theta(J))^{(2I-2)} \cdot D(J) = 1/(2I-1) \tag{17b}$$

for I = 2,3,···,N3

$$D[(N1+3)/2 - J] = D(J) \tag{17c}$$

for J = 2,3,···,N3

$$D(N1+1-J) = D(J) \tag{17d}$$

for J = 1,2,···,(N1 - 1)/2

$$D[(N1+1)/2] = 2D(1) \tag{17e}$$

b. Equations when (N1 + 1) is not a multiple of 4

Let θ(I) and N3 be defined as in Eqs. (3a) and (16). Then

$$\sum_{J=1}^{N3} 2D(J) + D(N3+1) = 2 \tag{18a}$$

$$\sum_{J=1}^{N3} (\cos\theta(J))^{(2I-2)} \cdot D(J) = 1/(2I-1) \tag{18b}$$

for I = 2,3,···,N3 + 1. Equations (17c), (17d), and (17e) remain unchanged.

29. In Eqs. (17a) and (17b) the I and J "subscripts" can be considered to designate the row and column for the coefficient matrix for N3 simultaneous linear equations in N3 unknowns. This also follows in Eqs. (18a) and (18b) except for an (N3 + 1) x (N3 + 1) system of linear equations. The system of equations for two examples are given below.

30. Let N1 = 7. Then (N1 + 1) = 8 is a multiple of 4, and

$$N3 = 2 \le (7+3)/4 = 2.5$$

$$\theta(1), \theta(2) = 0,\ 60^{\circ}$$

$$\cos(60) = 1/2.$$

Therefore Eqs. (17a) and (17b) imply that

$$D(1) + D(2) = 1$$

$$D(1) + D(2)/4 = 1/3.$$

These equations are satisfied by

$$D(1) = 1/9, \quad D(2) = 8/9.$$

The remaining equations, (16c), (16d), and (16e), imply that

$$D(1),\ D(2),\ \cdots,\ D(7) = 1/9,\ 8/9,\ 8/9,\ 2/9,\ 8/9,\ 8/9,\ 1/9.$$

31. Next, let N1 = 9. Than (N1 + 1) = 10 is not a multiple of 4, and

$$N3 = 3 \le (9 + 3)/4 = 3.0$$

$$\theta(1), \theta(2), \theta(3) = 0,\ 45^{\circ},\ 90^{\circ}$$

$$\cos(45^{\circ}) = 1/\sqrt{2}.$$

Therefore, Eqs. (18a) and (18b) imply that

$$2D(1) + 2D(2) + D(3) = 2$$

$$D(1) + D(2)/2 = 1/3$$

$$D(1) + D(2)/4 = 1/5.$$

The solution to these equations is

$$D(1),\ D(2),\ D(3) = 1/15,\ 8/15,\ 4/5.$$

The remaining equations, (17c), (17d), and (17e), imply that

$$D(1),\ D(2),\ \cdots,\ D(9) = 1/15,\ 8/15,\ 4/5,\ 8/15,\ 2/15,\ 8/15,\ 4/5,\ 8/15,\ 1/15.$$

32. Reference (a) gives a more theoretical approach to the methods of integration and describes another method for determining the factors D(I) using a polynomial approach. This method was compared with the direct inversion of the Eqs. (16) and (17). The results indicated that the direct inversion method was more stable numerically and took less computer time.

Units

33. All the programs use a mixed system of units. In general, the magnetic units are in the cgs system, e.g., the magnetic field intensity $\overline{H}$ ($f(\theta,\varphi)$ in Eq. (5)) is in gammas (10^{-5} oersted with a permeability of one). The units for a multipole moment of degree n is pole-centimeter^n, e.g., monopole moment is in poles, dipole moment is in pole-centimeters (cm), quadrupole moment is in pole-cm^2. The unit pole is equivalent to gauss-cm^2. The unit of length is inches. This is used for the measurement radius R1 and the multipole position vectors P (for programs SA4024 and SA5024). The unit of gammas is used for the data array F(I,J) and for the error parameters EG and ED (for programs SA4024 and SA5024). Any parameters that represent angles or angular errors are in degrees.

Sample Problems

34. Appendix F contains the input and output data for SA3024 using the same measurement data as in SA2024 (Figures 5 and 6). The file BN3024, used in the execution, is the binary version of SA3024. Only the first 16 binary records (subprograms) are used when executing via INTERCOM. As before, the example was executed in two different ways on the INTERCOM System. The problem was also submitted to BATCH processing via INTERCOM to demonstrate a method of using the plotting option of the program. One of the two identical plots that resulted from the BATCH processing is included in Figure 9. Although the example in Section II of Appendix F was set up to use the GOULD electrostatic plotter, the data was later plotted on the CALCOMP to simplify reproduction problems.

35. Several characteristics of the analysis can be observed by comparing Appendices D and F and Figures 6 and 9. The first characteristic is the data processing. This involves the conversion of the data units and the adjustment of the curves so that all the curves begin and end as near as possible to the beginning and ending of the first curve. The curve adjustment can be observed by comparing the print out of the data tape on page D-1 and the data printed on page D-2. The first points of curves 1 and 2 on the data tape are -504.3 and -902.8. After the data processing these points are -504.3 and -502.8. Another data adjustment in SA3024 has to do with the monopole moment. Since the monopole moment should be zero if the data has absolute accuracy, then the data is adjusted to produce this condition. The procedure involves the computation of the monopole moment, and then, the subtraction of the magnetic field of the monopole component from the data. A comparison of the data printed on page D-2 and on pages F-2 and

F-3 show a difference of about 24 gammas. This appears as the monopole moment of 14.3··· printed on page F-2. The data plotted in Figure 9 represents the data that would have been measured if absolute accuracy was attainable.

36. An example of the method of printing out the spherical harmonic coefficients is shown on page F-3. For each nth degree harmonic term, the coefficients A_n^m for $m = 0,1,\cdots,n$ are printed on the first line, and the coefficients B_n^m for $m = 1,2,\cdots,n$ are printed on the second line. It should be noted that in the definitions related to Eqs. (5) and (6), the coefficients A_n^m and B_n^m are coefficients for the Schmidt polynomials and not for the associated Legendre polynomials. The relationship is discussed in more detail in Appendix K.

PROGRAM SA4024

Introduction

37. The program SA4024, listed in Appendix G, was devised to assist in the conduct of error studies relating to the analysis of satellite magnetism. The program is set up to generate data simulating measurements around a specified system of multipole magnets representing the satellite. Simulated errors can be inserted into the generated data to represent position and instrumentation errors. The generated data is written on a data file (DAT024) in the same format as the one used in the data acquisition procedures with the punched paper tape. The data file is generated in a form that can be used with programs SA2024 and SA3024. The program SA4024 also contains the plotting option.

38. Special subprograms used in the program perform such tasks as:

a. The computation of the magnetic field vector at a remote location from a multipole magnet

b. The generation of values of associated Legendre polynomials and Schmidt functions

c. The interpolation and plotting of the magnetic data.

Input Data

39. The input data for SA4024 varies considerably from the preceding programs since it includes specifications for multipole magnets and data errors. The data is entered in the following order:

a. First line of data or data card - format: (4I5)

NO - The number of curves of data (NO is even and ≤ 16.)

N1 - The number of equally spaced data points per curve from 0 thru 360 degrees colatitude (N1 is odd and ≤ 33. The first and last data points for each curve correspond to measurements at 0 degrees colatitude.)

NH - The total number of different harmonics (degrees) of multipole magnets to be considered

IP - The parameter that determines whether or not the data is to be interpolated and plotted ($IP \neq 0$ means that the data will be interpolated and plotted.)

b. Second line of data or data card - format: (5F8.4)

R1 - The radius of the measurement sphere in inches

EG - The error (in gamma) to be randomly inserted into the data to represent instrumentation inaccuracies

EA - The angular error (in degrees) to be randomly inserted into the data to represent measurement position errors

ED - The constant error (in gammas) to be inserted into the data to represent offset in the instrumentation. (This will be analyzed as monopole moment.)

PY - The scale factor (gammas/inch) for the y-axis if the data is to be plotted. (If PY = 0.0 a factor will be computed from the data.)

c. Third line of data or data card - format: (7A10)

F9 - The format for reading and printing the spherical coefficients A(I) and B(I), e.g., (1H ,7E10.4)

All of the following data is repeated "NH" times:

d. Next line of data or data card - format: (2I5)

NN - The harmonic number (degree) of the multipole data being read in (NN = 1 for dipoles, 2 for quadrupole, etc.)

NM - The number of multipoles with harmonic number NN

The following data is repeated "NM" times:

e. Next lines of data or data cards - format: (3F8.4)

P - The position vector (in inches) of the multipole in rectilinear coordinates (P_x, P_y, P_z.)

f. Next lines of data or data cards - format: F9

A(I) - The spherical coefficients A_{NN}^{I-1} for the multipole of degree NN where I = 1,2,···,NN + 1 (The order of the Ith coefficient is I - 1.)

g. Next lines of data or data cards - format: F9

B(I) - The spherical coefficients B_{NN}^{I} for the multipole of degree NN where I = 1,2,···,NN (The order of the Ith coefficient is I.).

(The relationships between spherical and rectilinear coefficients for dipoles and quadrupoles are given in Eqs. (9) and (10).)

Output Data

40. The program computes data representing measurements of the specified system of multipole magnets. There are NO curves of data computed, each containing N1 data points. The computations are made by using Eq. (7). For instance assume that the system has NM multipoles of degree N and that $f_{nj}(\theta,\varphi)$ represents (as in Eq. (7)) the normal component of the magnetic field from the jth multipole of degree N. Then the total magnetic field $f_n(\theta,\varphi)$ for all multipoles of degree N is

$$f_n(\theta,\varphi) = \sum_{j=1}^{NM} f_{nj}(\theta,\varphi) \tag{19}$$

If NH is the total number of different degrees for the system of multipoles then the normal component of the magnetic field for the total system is

$$f(\theta,\varphi) = \sum_{N=1}^{NH} f_n(\theta,\varphi) . \tag{20}$$

The program sets up the data array F(I,J) in this manner. (F(I,J) is defined in Eq. (2).) The data array is then written onto the data file DAT024. The file can then be used with either program SA2024 or program SA3024. A data tape can also be made of this file by listing the file under SYSTEM/BASIC with the tape punch unit on.

Sample Problems

41. SA4024 was used to compute the data for several examples. These are listed in Appendix H and in Figures 10 through 24. BN4024 is the binary version of SA4024. When executing the program via INTERCOM only the first nine binary records are used. The first section in Appendix H presents the input and output data for the sample problem illustrated in Figure 5. The instrumentation inaccuracy was assumed to be ± 1.0 gamma by setting EG = 1.0. The positions for the measurements of the data were considered to have inaccuracies of ± 0.2 degrees (EA = 0.2). The set of data curves were assumed to be offset by 25.0 gammas (ED = 25.0). The data curves representing the simulated measurements are listed on page H-3. Section II of Appendix H contains the data file DAT024 that was generated with this data in a format like the data tape. In Section III of Appendix H the sample problem and several other problems were submitted to BATCH. The resulting data was also plotted. The data in Figure 10 for the sample problem is nearly identical to the data in Figure 9. The remaining examples in section III represent the individual and combined data for the first eight spherical harmonics ($n = 1,2,\cdots\cdots,8$) of the sample problem. These will be discussed in more detail in later paragraphs.

42. SA4024 was also used to show data curves for individual components of dipole and quadrupole moments. These are included in Figures 11 through 24. A small diagram is also included on each figure to illustrate the particular component that is represented by the curves. Curves are shown for components in both spherical and rectilinear coordinates according to Eqs. (9) and (10).

PROGRAM SA5024

Introduction

43. The program SA5024, listed in Appendix I, was devised to simplify the error studies. It performs functions similiar to both programs SA3024 and SA4024, i.e., SA5024 is both a data generation and a data analysis program. A system of multipole magnets is specified in the input data to represent the satellite as in SA4024. Since the data is analyzed as it is generated it is not written onto a data file as in SA4024. However, the program does include the plotting option when executing in the BATCH mode. Also, the integration option discussed in paragraphs 25 through 32 is included. The primary subprograms in SA5024 perform most of the functions already described in the sections on SA3024 and SA4024.

Input Data

44. The input data for SA5024 resembles the input data for the program SA4024 since it consists mostly of specifications for multipole magnets and data errors. The data is entered in the following order:

a. First line of data or data card - format: (7I5)

NO - The number of curves of data (NO is even and $\leq$ 16.)

N1 - The number of equally spaced data points per curve from 0 through 360 degrees colatitude (N1 is odd and $\leq$ 33. The first and last data points for each curve correspond to measurements at 0 degrees colatitude.)

NH1 - The total number of different harmonics (degrees) of multipole magnets to be considered

NH2 - The harmonic number (degree) representing the highest degree spherical harmonic term to be computed from the data (NH2 = 1 for dipoles, 2 for quadrupoles, etc.)

IP1 - The parameter that determines whether or not the data is to be interpolated and plotted (IP1 $\neq$ 0 means that the data will be interpolated and plotted.)

IP2 - The parameter that determines whether or not the magnetic data is to be printed ($IP2 \neq 0$ means that the data will be printed.)

IW - The parameter that determines which integrating scheme is to be used ($IW = 0$ means that the exact, algebraic scheme is to be used.)

b. Second line of data or data card - format: (5F8.4)

R1 - The radius of the measurement sphere in inches

EG - The error (in gammas) to be randomly inserted into the data to represent instrumentation inaccuracies

EA - The angular error (in degrees) to be randomly inserted into the data to represent measurement position errors

ED - The constant error (in gammas) to be inserted into the data to represent offset in the instrumentation (this will be analyzed as monopole moment)

PY - The scale factor (gammas/inch) for the y-axis if the data is to be plotted (if $PY = 0.0$ a factor will be computed from the data.)

c. Third line of data or data card - format: (7A10)

F9 - The format for reading and printing the spherical coefficients A(I) and B(I), e.g., (1H ,7E10.4)

All of the following data is repeated "NH" times:

d. Next line of data or data card - format: (2I5)

NN - The harmonic number (degree) of the multipole data being read in ($NN = 1$ for dipoles, 2 for quadrupole, etc.)

NM - The number of multipoles with harmonic number NN

The following data is repeated "NM" times:

e. <u>Next lines of data or data cards - format: (3F8.4)</u>

P - The position vector (in inches) of the multipole in rectilinear coordinates (P_x, P_y, P_z)

f. <u>Next lines of data or data cards - format: F9</u>

A(I) - The spherical coefficients A_{NN}^{I-1} for the multipole of degree NN where I = 1,2,···,NN + 1 (The order of the Ith coefficient is I - 1.)

g. <u>Next lines of data or data cards - format: F9</u>

B(I) - The spherical coefficients B_{NN}^{I} for the multipole of degree NN where I = 1,2,···,NN. (The order of the Ith coefficient is i.).

(The relationships between spherical and rectilinear coefficients for dipoles and quadrupoles are given in Eqs. (9) and (10).)

Output Data

45. Initially, the program SA5024 computes the simulated measurement data as in Eqs. (19) and (20). Next, the spherical harmonic coefficients are computed according to Eq. (6). The program also computes the exact dipole moment from the multipole specifications. This value is compared with the dipole moment computed from the simulated measurement data. The percent error is printed as part of the output data.

Sample Problems

46. Appendix J contains the input and output data for several problems using SA5024. As in the other programs the binary version of SA5024 is the file BN5024. When executing the program via INTERCOM, only the first 15 binary records are used. In the first section of Appendix J an analysis was conducted of the sample problem illustrated in Figure 5. The parameters EG, EA, and ED were all set to zero (page J-1) so that no simulated errors were inserted into the data.

47. Figure 25 shows eight curves of the computed data for the three dipoles in the sample problem. Actually, 16 curves, each containing 33 data points, were used in the computations (based on Eq. (12) and Table 1). These values for the

parameters NO and N1 allowed the computation of the spherical harmonic coefficients for the first eight multipole components. The program SA4024 was used to compute eight data curves for each multipole component to demonstrate the type of curves representing each component. These are presented in Figures 26 through 33. These were also combined and plotted by SA4024. Figure 34 displays the combined data. The first data curve (xz-plane) was chosen to further demonstrate the characteristics of spherical harmonic approximation. Figure 35 contains four curves. The first curve is the original data from the sample problem. Curve 2 represents the dipole component. Curve 3 represents the combination of dipole and quadrupole components. Curve 4 represents the combination of the first eight multipole components. The convergence of the series in Eq. (7) is not obvious from the coefficients computed in Section I of Appendix J. Table 2 summarizes the peak coefficients and the data peaks for each degree n. Although the magnitude of the peak coefficient increases with increasing n, the data peak (magnetic field at 96 inches) decreases. In fact, the first eight multipole components account for about 97 or 98 percent of the original data curves. This indicates that higher degree components are relatively insignificant.

TABLE 2

PEAK DATA FOR THE MULTIPOLE COMPONENTS OF THE SAMPLE PROBLEM

Degree n	$\left(\max_{0 \le m \le n} \{ \lvert A_n^m \rvert, \lvert B_n^m \rvert \} \right)$	Data Peak (for 8 curves)
1	10^4	13.79
2	$-.9472\text{X}10^7$	803.79
3	$.2341\text{X}10^{10}$	839.34
4	$.1545\text{X}10^{12}$	367.51
5	$-.2812\text{X}10^{14}$	211.03
6	$+.4533\text{X}10^{16}$	189.44
7	$+.3541\text{X}10^{18}$	57.51
8	$-.4517\text{X}10^{20}$	35.47
Original		1817.94
Sum of Components		1786.61

48. Section II of Appendix J demonstrates the procedure for submitting the program SA5024 for BATCH execution. The data specifies two problems consisting of

offset dipoles. In the first problem the dipole is aligned parallel to the x-axis and offset along the x-axis. The resultant curves are presented in Figure 36. The second problem is a dipole aligned parallel to the y-axis and offset along the x-axis. These curves are presented in Figure 37. A small diagram is included in each figure to demonstrate the geometric configuration.

49. A small error study was made to demonstrate the relationship between the accuracy of the dipole analysis; the parameters NO, N1, and R1; and the measurement errors (represented by EG and EA). The sample problem in Figure 5 was again selected. This example has a resultant dipole moment of 1000 gauss-cm^3, but the magnetic field from this dipole component is embedded in the large field from quadrupole and higher terms. Curve 2 in Figure 35 demonstrates this condition. Also Table 2 shows that the peak of the dipole curves is only 13.79 gammas out of a total of 1817.94 gammas for the total system. Table 3 contains a summary of the study. Several different values of NO, N1, IW, R1, EG and EA were used. The indications are that increasing R1 doesn't help much if EA and EG are too large. Also, there isn't much improvement in the accuracy of the computations when using the exact integration scheme (i.e., when setting IW = 0). Probably the most significant observation is that the parameter EA, representing position errors, has a much greater effect on the analysis accuracy than the parameter EG, representing instrumentation errors.

TABLE 3

MAXIMUM ERROR VERSUS PARAMETRIC VARIATIONS IN COMPUTING THE DIPOLE MOMENT FOR THE SAMPLE PROBLEM

Dipole Moment Error (gauss-cm^3)

	Parameters IW, R1, EG, EA						
NO, N1	1,96,0,0	1,96,1,1	1,96,5,2	1,72,1,1	1,72,5,2	1,120,1,1	0,72,1,1
6, 13	-128 (x)	-135 (y)	-239 (y)	-2450 (x)	-2510 (x)	868 (x)	-2430 (x)
8, 17	-.63 (x)	653 (y)	1320 (y)	1760 (y)	3590 (x)	395 (y)	1780 (y)
4, 25	431 (x)	887 (x)	1470 (y)	7420 (x)	8390 (x)	500 (y)	7440 (x)
6, 25	-54 (x)	-734 (y)	1440 (y)	-1160 (y)	-2170 (y)	-462 (y)	-1160 (y)
8, 25	6.3 (x)	-407 (x)	-785 (x)	220 (y)	-505 (x)	-298 (x)	223 (y)
10, 25	-2.8 (x)	-515 (x)	-1030 (x)	-1080 (x)	-2200 (x)	-306 (x)	-1080 (x)
12, 25	-2.7 (x)	-884 (x)	-1730 (x)	-1620 (x)	-3250 (x)	-570 (x)	-1620 (x)
Minimum detectable dipole based on R1 & EG		73	363	31	153	142	31

NO, N1, IW, R1 = 12, 25, 1, 96

EG	ERROR
8	-66 (x)
4	23 (x)
2	-6.8 (x)
1	-10.9 (y)
.5	-2.3 (y)
.25	-2.8 (x)
0	-2.7 (x)

EA	ERROR
4	1003 (y)
2	-854 (x)
1	505 (y)
.5	362 (x)
.25	-197 (x)
.125	56 (x)
0	-2.7 (x)

Note: An error of -1620 gauss-cm^3 means that the program computed a dipole moment of -620 gauss-cm^3 instead of +1000 gauss-cm^3. The letter contained in parentheses indicates the dipole component containing the error.

Chapter 4

PRINCIPAL SUBPROGRAMS

INTRODUCTION

50. The programs described in the preceding sections were constructed to utilize several algorithms which were incorporated into functions and subroutines. These include algorithms for computing the spherical harmonic coefficients, computing the vector magnetic field emanating from a multipole magnet, computing the associated Legendre Polynomials, computing the Schmidt polynomials, and interpolating and plotting data curves. The subprograms have been code in a manner which makes them useful as building blocks for other programs. They are discussed separately in the following paragraphs.

SUBROUTINE AMPMNT

51. The subroutine AMPMNT is used to compute the spherical coefficients A_n^m and B_n^m for the data array F(I,J). The computation is based on Eq. (6). The subroutine is constructed to compute the coefficients for only one degree at a time. The calling sequence is

```
CALL AMPMNT (NO, N1, N, R1, F, P, A, B)
```

where the arguments are defined as follows:

a. Input

NO - The number of curves of data

N1 - The number of equally spaced data points per curve from 0 through 360 degrees colatitude

N - The degree of the spherical harmonic coefficients to be computed

R1 - The radius of the measurement sphere in inches

F(IJ) - The measured data for $IJ = I + (J - 1) \cdot N1$, $I = 1,2,\cdots,N1$, and $J = 1,2,\cdots,NO$ (The data is stored in a one-dimensional array to conserve storage, i.e., equating the one-dimensional array to the two-dimensional array in Eq. (6) gives $F(IJ) \equiv F(I,J)$.)

P(I) - The first "(N1 + 1)/2" weighting factors Y(I) in Eq. (6), i.e., $P(I) = Y(I)$ for $I = 1,2,\cdots,(N1 + 1)/2$.

b. Output

A(M + 1) - The spherical coefficients A_n^m for $M = 0,1,2,\cdots,N$ (see Eq. (8))

B(M) - The spherical coefficients B_n^m for $M = 1,2,3,\cdots,N$ (see Eq. (8)).

52. There are several other subroutines that are used to support AMPMNT in addition to the standard FORTRAN IV machine routines. AMPMNT internally calls a subroutine labeled POLVAL which in turn uses functions labeled SPNM, PNM, and ANF. These routines are used to compute the appropriate values for the Schmidt polynomials $P_n^m(\cos(\theta(I)))$ in Eq. (6). Also, the weights P(I) are set up externally by special subroutines before being input to the AMPMNT subroutine. This process requires the use of subroutines labeled WGT1, WGT2, WGT3, and GAUSEL. Listings of all of these subroutines are included in Appendix I.

SUBROUTINE AMPFLD

53. The subroutine AMPFLD is used to compute the magnetic field at the point $\bar{R}$ from a multipole magnet of degree n centered at the point $\bar{P}$. The magnet is specified in terms of the spherical harmonic coefficients A_n^m and B_n^m. The calling sequence is

CALL AMPFLD (R, P, N, A, B, F)

where the arguments are defined as follows:

a. Input

R(I) - The rectilinear coordinates (in inches) of the point at which the field is to be computed (R(1) = x-coor., R(2) = y-coor., and R(3) = z-coor.)

P(I) - The rectilinear coordinates (in inches) of the center position of the multipole magnet (P(1) = x-coor., P(2) = y-coor., and P(3) = z-coor.)

N - The harmonic degree of the multipole magnet

A(M + 1) - The spherical coefficients A_n^m for M = 0,1,2,···,N (see Eq. (8))

B(M) - The spherical coeficients B_n^m for M = 1,2,···,N (see Eq. (8))

b. Output

F(I) - The total magnetic field in rectilinear coordinates (H_x = F(1), H_y = F(2), and H_z = F(3)).

54. The subroutine computes the total magnetic field vector for a single multipole magnet. The computations are based on relationships that are similiar to Eq. (7) except that only a single multipole magnetic (N is fixed) is considered. Also, the subroutine gives all three rectilinear components of the magnetic field instead of just the radial component. The relationships are presented below using terminology that can be found in references (b) and (c). Let $\bar{R}_1$ be defined as the vector from the magnet to the point $\bar{R}$, i.e.,

$$\bar{R}_1 = \bar{R} - \bar{P} \tag{21}$$

where $\bar{R}$ and $\bar{P}$ are as defined above. Next, let

$$\bar{R}_1 = (R_1, \varphi, \theta) \tag{22}$$

be the representation of $\bar{R}_1$ in spherical coordinates where R_1 is the vector length, θ is the colatitude, and φ is the longitude relative to a system of coordinates centered at the multipole position. Now, define an orthonormal set of vectors $\bar{r}$, $\bar{\theta}$, and $\bar{\varphi}$ in spherical coordinates as

$$\bar{r} = \cos\varphi \sin\theta \,\bar{i} + \sin\varphi \sin\theta \,\bar{j} + \cos\theta \,\bar{k} \tag{23a}$$

$$\bar{\theta} = \cos\varphi \cos\theta \,\bar{i} + \sin\varphi \cos\theta \,\bar{j} - \sin\theta \,\bar{k} \tag{23b}$$

$$\bar{\varphi} = -\sin\varphi \,\bar{i} + \cos\varphi \,\bar{j} \tag{23c}$$

where $\bar{i}$, $\bar{j}$, and $\bar{k}$ represent the orthonormal set of vectors along the x, y, and z-axis, respectively (see Figure 1). It can be seen that

$$\bar{r} = \bar{P}_1/R_1. \tag{24}$$

Then, the total field vector $\bar{H}_n(\bar{R})$ for the nth degree multipole is given as

$$\bar{H}_n(\bar{R}) = H_{nr}(\bar{R})\bar{r} + H_{n\theta}(\bar{R})\bar{\theta} + H_{n\varphi}(\bar{R})\bar{\varphi} \tag{25}$$

where

$$H_{nr}(\bar{R}) \equiv \left[(n + 1)/R_1^{\,n + 2}\right] \sum_{m=0}^{n} (A_n^m \cos m\varphi + B_n^m \sin m\varphi)P_n^m(\cos \theta) \tag{26a}$$

$$\begin{aligned} H_{n\theta}(\bar{R}) \equiv (1/R_1^{\,n + 2}) \sum_{m=0}^{n} (A_n^m \cos m\varphi + B_n^m \sin m\varphi)\Big[C_n^m P_n^{m + 1}(\cos \theta) \\ -m \cos \theta \; P_n^m(\cos \theta)/\sin \theta\Big] \end{aligned} \tag{26b}$$

$$H_{n\varphi}(\bar{R}) \equiv (1/R_1^{\,n + 2}) \sum_{m=0}^{n} m(A_n^m \sin m\varphi - B_n^m \cos m\varphi)P_n^m(\cos \theta)/\sin \theta. \tag{26c}$$

The coefficient C_n^m, in Eq. (26b), has the value

$$\begin{aligned} C_n^m &= \left[(n - m)(n + m + 1)\right]^{\frac{1}{2}} \text{ for } (m > 0) \\ &= \left[n(n + 1)/2\right]^{\frac{1}{2}} \text{ for } (m = 0). \end{aligned}$$

Also, the terms which have sin θ as a divisor are computed using the following identities:

$$\begin{aligned} 2mP_{n,m}(\cos \theta)/\sin \theta &= (n - m + 1)(n - m + 2)P_{n+1,m-1}(\cos \theta) + P_{n+1,m+1}(\cos \theta) \\ &\quad \text{for } (m > 0) \\ &= 0 \text{ for } (m = 0) \end{aligned} \tag{27}$$

$$\begin{aligned} P_n^m(\cos \theta) &= \left\{2 \frac{(n - m)!}{(n + m)!}\right\}^{\frac{1}{2}} P_{n,m}(\cos \theta) \text{ for } (m > 0) \\ &= P_{n,0}(\cos \theta) \text{ for } (m = 0). \end{aligned} \tag{28}$$

The function $P_{n,m}(\cos\theta)$ represents the associated Legendre polynomial of degree n and order m. $P_n^m(\cos\theta)$ represents the Schmidt polynomial of degree n and order m. These polynomials are discussed in more detail in Appendix K. The radial component $f_n(\varphi,\theta)$ of the magnetic field at the point $\bar{R}$ is computed from the total field $\bar{H}_n(\bar{R})$ as

$$f_n(\varphi,\theta) = \bar{H}_n(\bar{R}) \cdot \bar{R}/(\bar{R} \cdot \bar{R})^{\frac{1}{2}}. \tag{29}$$

55. The subroutine AMPFLD uses several external subprograms including SPNM, PNM, ANF, SPCOOR, and SUM. These are listed in Appendix I. The subprograms SPNM, PNM, and ANF are used to compute the Schmidt and associated Legendre polynomials. The subroutines SPCOOR and SUM are used in mathematical operations with vectors. SPCOOR transforms a vector from rectilinear to spherical coordinates. SUM multiplies vectors and scalars, and then, adds the products.

FUNCTIONS PNM, SPNM, AND ANF

56. The function subprograms PNM, SPNM, and ANF are used in the generation of values for spherical polynomials. PNM computes the value of the associated Legendre polynomial $P_{n,m}(x)$ of degree n and order m at the point x for $|x| \le 1$. (If m = 0 the subprogram computes the value of the regular Legendre polynomial.) SPNM converts this value into the Schmidt function $P_n^m(x)$. ANF simply computes the factorial value for the integer N. Let y represent a variable used in a computer program which is to be set equal to a polynomial value. Then the calling sequence for the subprograms is defined as follows:

$y = P_n(x)$ is written as Y = PNM(N,0,X) for $N \ge 0$ and $|X| \le 1$

$y = P_{n,m}(x)$ is written as Y = PNM(N,M,X) for $N \ge 0$, $M \ge 0$, and $|X| \le 1$

$y = P_n^m(x)$ is written as Y = SPNM(N,M,X) for $N \ge 0$, $M \ge 0$, and $|X| \le 1$.

The factorial subprogram is used as follows:

$y = n!$ is written as Y = ANF(N) for $N \ge 0$.

The development and use of these subprograms was reported in more detail in an internal technical note. This is included in Appendix K.

SUBROUTINE DATPLT

57. The subroutine DATPLT is used to interpolate and plot the curves of data. It is written to make use of either the GOULD electrostatic plotter or the CALCOMP 570 pen plotter. Most of the data curves in the enclosed figures were plotted on the CAPCOMP plotter. Figure 38 shows a set of data curves plotted on the GOULD plotter. These correspond to the curves in Figure 6. The routine sets up x and y-coordinate arrays for the magnetic field data F(IJ). A plotting symbol (the curve number) is used to mark each data point along the data curves. The data is plotted as magnetic field versus the angle of colatitude as shown in Figure 6. The calling sequence is

CALL DATPLT (NO, N1, F, YDIST)

where the argument list is defined of follows:

NO - The number of curves of data

N1 - The number of data points per curve

F(IJ) - The array of data curves for $IJ = J + (I - 1)\cdot N1$; $J = 1,2,\cdots,N1$; and $I = 1,2,\cdots,NO$

YDIST - The value in gamma to be used for one inch of the y-axis (If YDIST = 0.0 the subroutine PKY is used to determine a suitable scale for the y-axis.).

58. The subroutine uses the external subprograms CALCM1, FNCTON, and PKY for execution. CALCM1 is contained in special plotting packages of subprograms that is in the NOL subroutine library. There are two separate packages that can be loaded when using the subroutines DATPLT and CALCM1. The package labeled GOULD1 is loaded when plotting with the GOULD electrostatic plotter. The package labeled CALCM1 is loaded when plotting with the CALCOMP plotter. Also a tape (TAPE99) must be loaded with the CALCM1 package. The subprograms FNCTON and PKY are listed in Appendix I. FNCTON computes the Fourier coefficients for each curve of data. These are used to increase the point density of the data to be plotted to one point for every five degrees of colatitude. This turns out to be 12 points per inch on the graphs or a total of 73 points per curve. The subprogram PKY is used to determine a suitable scale for the y-axis if YDIST is zero.

SUBROUTINE FNCTON

59. The subroutine FNCTON is used to perform a Fourier analysis on a single-dimensioned array of data that represents the values along a curve at equally spaced intervals. The program is used primarily for interpolating data. The calling sequence is

CALL FNCTON (F, N, C, CO, PHI, N2, DEV)

where the argument list is defined as follows:

a. Input

F(I) - The data array representing the values of the curve at N equally space values $\theta(I) = 2\pi(I - 1)/N$ for $I = 1,2,\cdots,N$

N - The number of data points

b. Output

C(I) - The array of Fourier amplitudes for $I = 1,2,\cdots,N2$

CO - The zero degree Fourier amplitude

PHI(I) - The array of Fourier phase angles for $I = 1,2,\cdots,N2$

N2 - The number of terms in the approximating Fourier series

DEV - The maximum difference between the data points and the curve defined by the Fourier series.

60. The coefficients are obtained by the following equations:

$$C_0 \equiv (1/N) \sum_{I=1}^{N} F(I) \tag{30}$$

$$S1_j \equiv (2/N) \sum_{I=1}^{N} F(I) \cdot \sin\left[2\pi j(I - 1)/N\right] \tag{31a}$$

$$C1_j \equiv (2/N) \sum_{I=1}^{N} F(I) \cdot \cos\left[2\pi j(I - 1)/N\right] \tag{31b}$$

$$C_j \equiv (S1_j^2 + C1_j^2)^{\frac{1}{2}} \text{ for } j \neq N/2$$
$$\equiv (S1_j^2 + C1_j^2)^{\frac{1}{2}}/2 \text{ for } j = N/2 \qquad (32)$$

$$PHI_j \equiv \tan^{-1}(C1/S1) \qquad (33)$$

for $j = 1,2,\cdots,N/2 + 1$.

61. This results in the approximation

$$F(I) \approx C_0 + \sum_{j=1}^{N2} C_j \sin\left[2\pi j(I - 1)/N + PHI_j\right] \text{ for } I = 1,2,\cdots N. \qquad (34)$$

Any other set of N' equally spaced values which begin at $\theta(1) = 0$ can be computed by substituting N' in place of N in Eq. (34). DATPLT uses the value of 72 for N'.

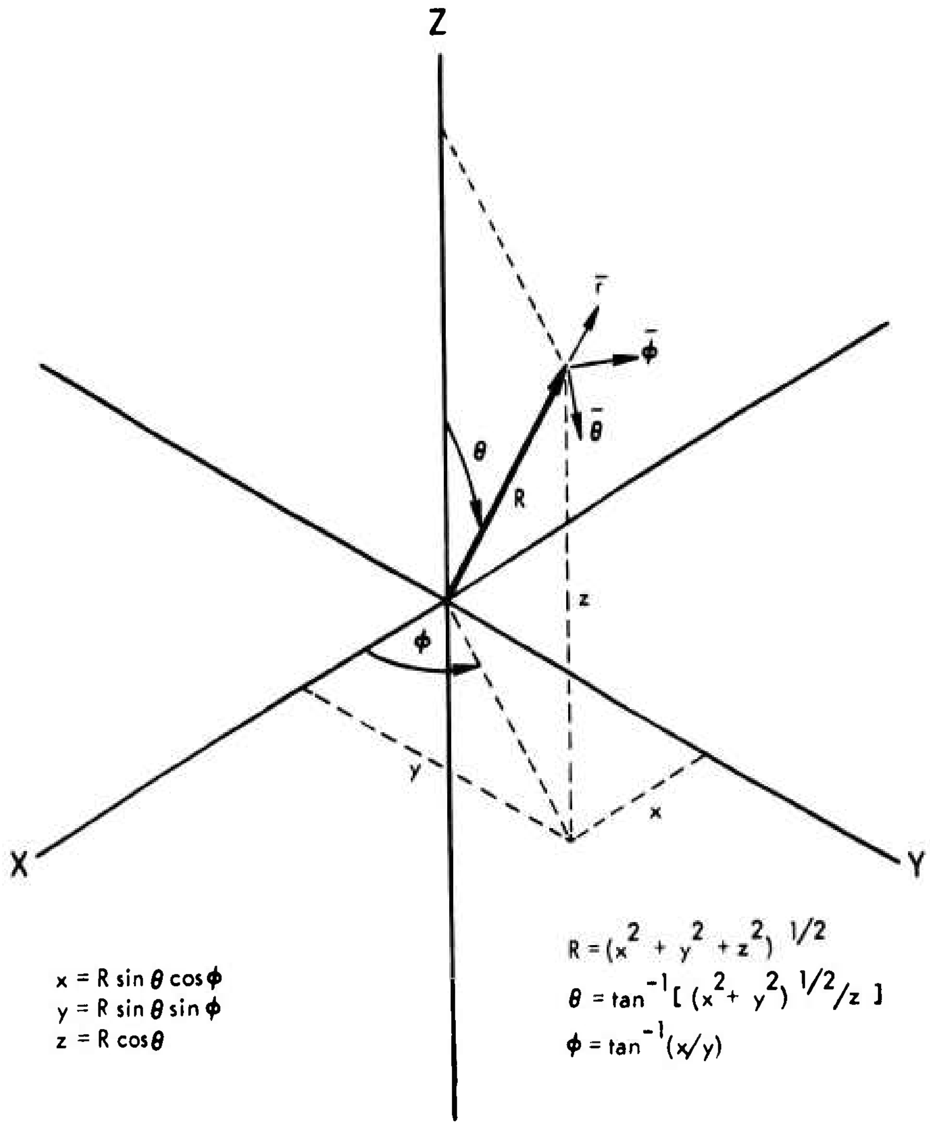
Chapter 5

CONCLUSIONS

62. The computer programs presented in this report are part of a major effort at NOL to develop and improve magnetic test procedures on spacecraft. The effort involves the development and evaluation of both data acquisition and data analysis procedures. The Spherical Field Coil Facility at NOL is being instrumented to handle both the acquisition and analysis of the test data. The proposed specifications for the instrumentation were presented i reference (d). More detailed specifications for the data acquisition and analysis system were forwarded to NRL as an enclosure to reference (e). The facility is scheduled to be operational by the beginning of fiscal year '75. A technical report describing the facility should be published shortly thereafter.

63. There are other analysis projects to be completed. One is the compilation and publication of a reference manual containing the mathematical equations for dipole and quadrupole computations. This will simplify slide-rule calculations which are made during testing. Another project involves the conduct of a more elaborate error analysis using the computer programs described above. This will be necessary to evaluate the minimum detectable dipole moment under several different conditions.

64. It should be noted here that there are several other methods for estimating the dipole moment of spacecraft. The most effective method to use in any particular case depends on a number of things including the size of the spacecraft, the availability of a gimballing system, the desired accuracy of the compensation, the complexity and permanence of the spacecraft's magnetism, etc. The procedure based on the spherical harmonic analysis of the spacecraft's magnetism has proven to be both fast and accurate for NRL satellites. Many of the tests have been completed in less than four hours and with an accuracy which allows dipole moment compensation of all but about 100 out of 6000 gauss-cm^3.



$x = R \sin\theta \cos\phi$
$y = R \sin\theta \sin\phi$
$z = R \cos\theta$

$R = (x^2 + y^2 + z^2)^{1/2}$
$\theta = \tan^{-1}[(x^2 + y^2)^{1/2}/z]$
$\phi = \tan^{-1}(x/y)$

FIG. 1 ILLUSTRATION OF SPHERICAL POLAR COORDINATES

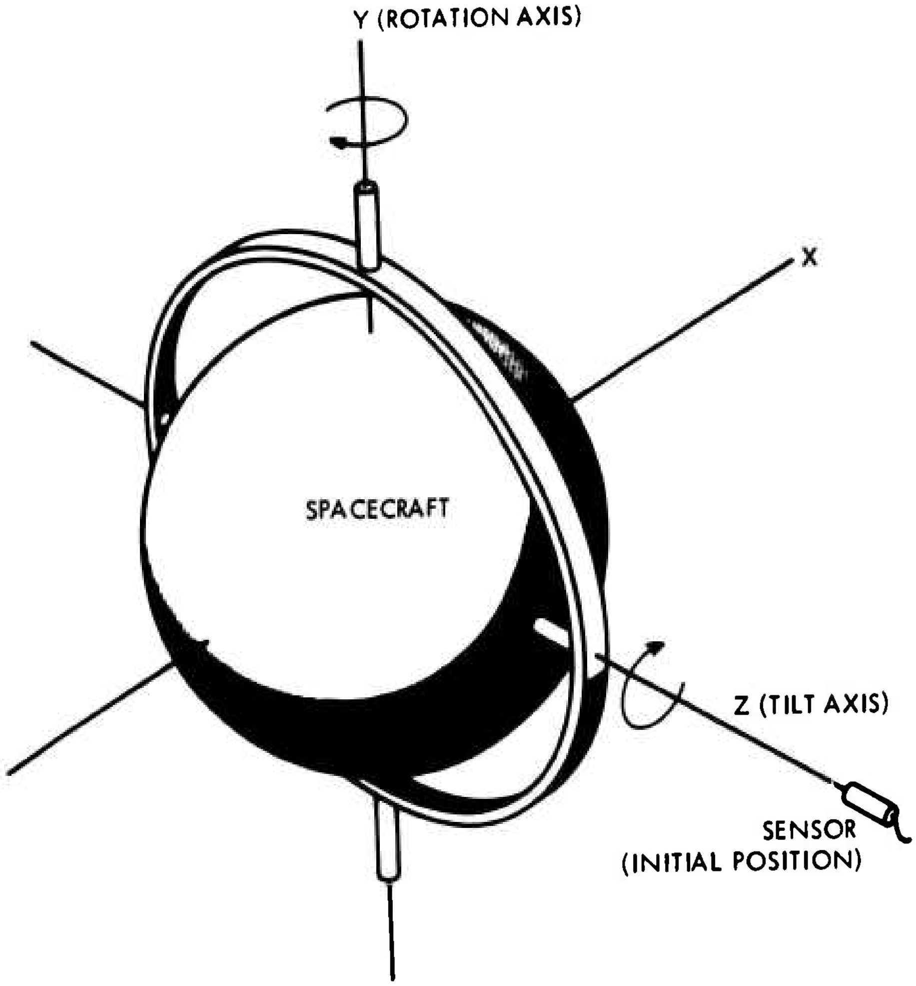


FIG. 2 DEGREES OF FREEDOM FOR MEASUREMENTS

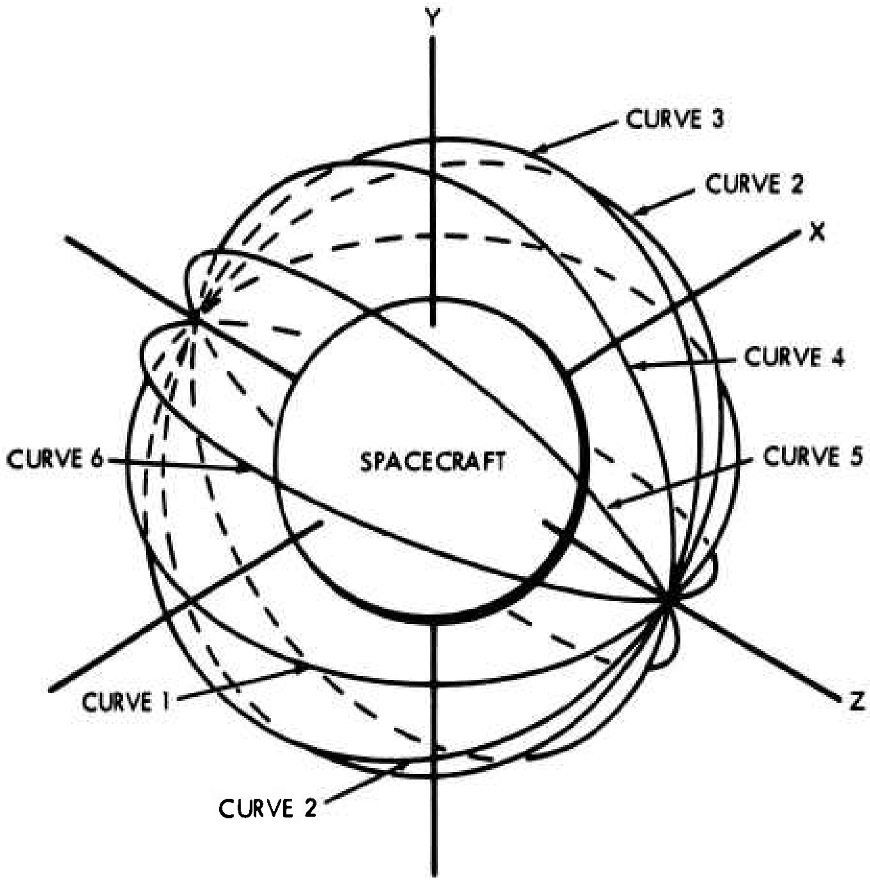
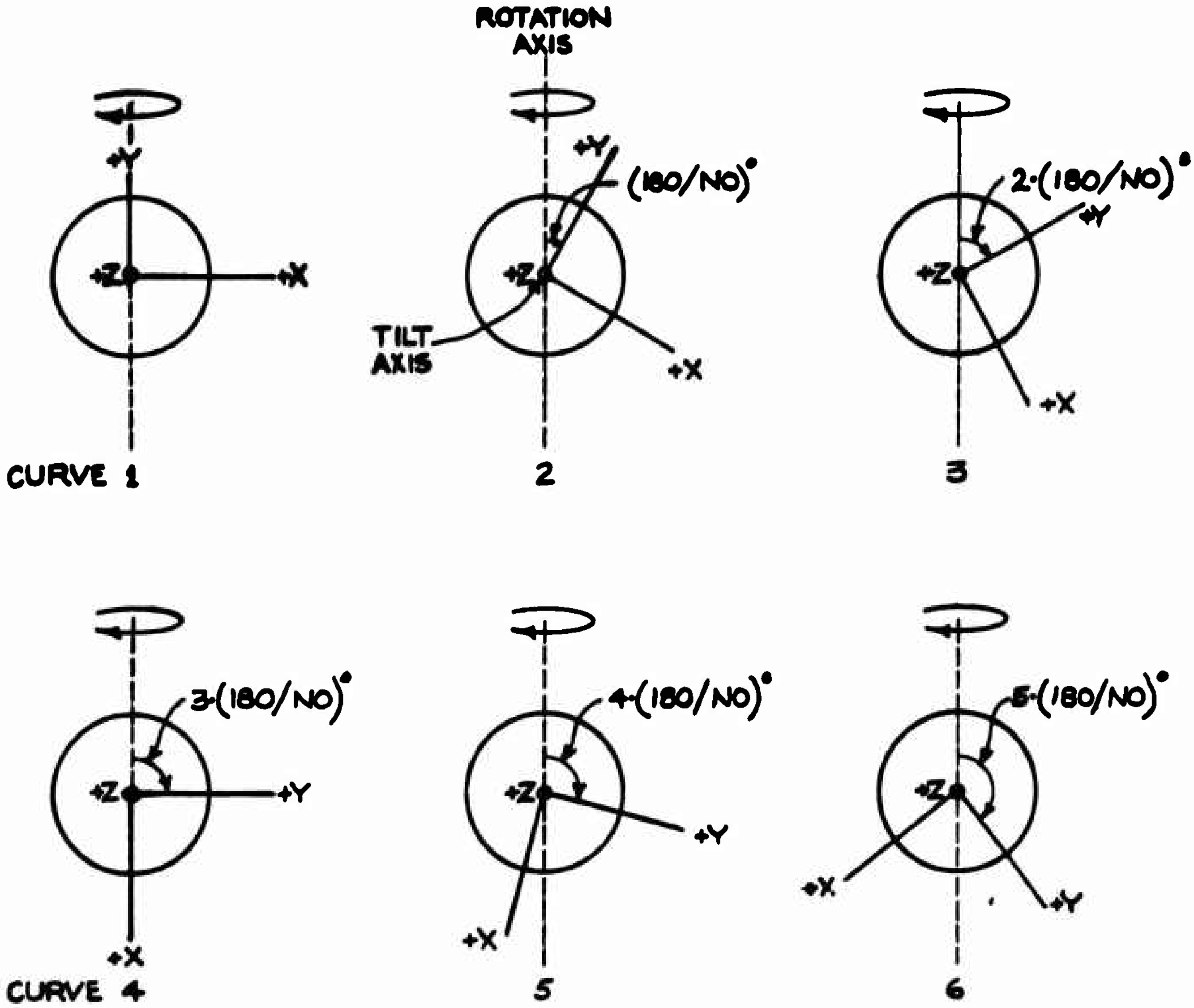


FIG. 3 SPHERE OF MEASUREMENTS

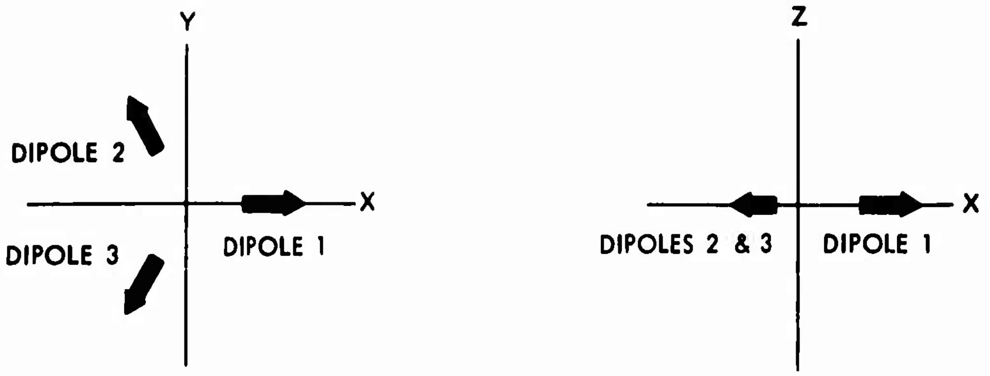


Notes:

1. The diagrams are viewed from the sensor position.
2. The turntable rotation is clockwise.
3. Each curve begins with +z-axis directed at the sensor.

FIG. 4 SATELLITE INITIAL POSITIONS FOR NO = 6
(Viewed from sensor position)

DIAGRAM OF DIPOLES



PARAMETRIC VALUES

PARAMETER	COMP.	DIPOLE		
		1	2	3
POSITION VECTOR	X	39.000	-21.000	-21.000
	Y	0.000	36.373	-36.373
	Z	0.000	0.000	0.000
DIPOLE MOMENT	D_x (A(2))	31,000.	-15,000.	-15,000.
	D_y (B(1))	0.	25,980.	-25,980.
	D_z (A(1))	0.	0.	0.

FIG. 5 ILLUSTRATION OF SAMPLE PROBLEM

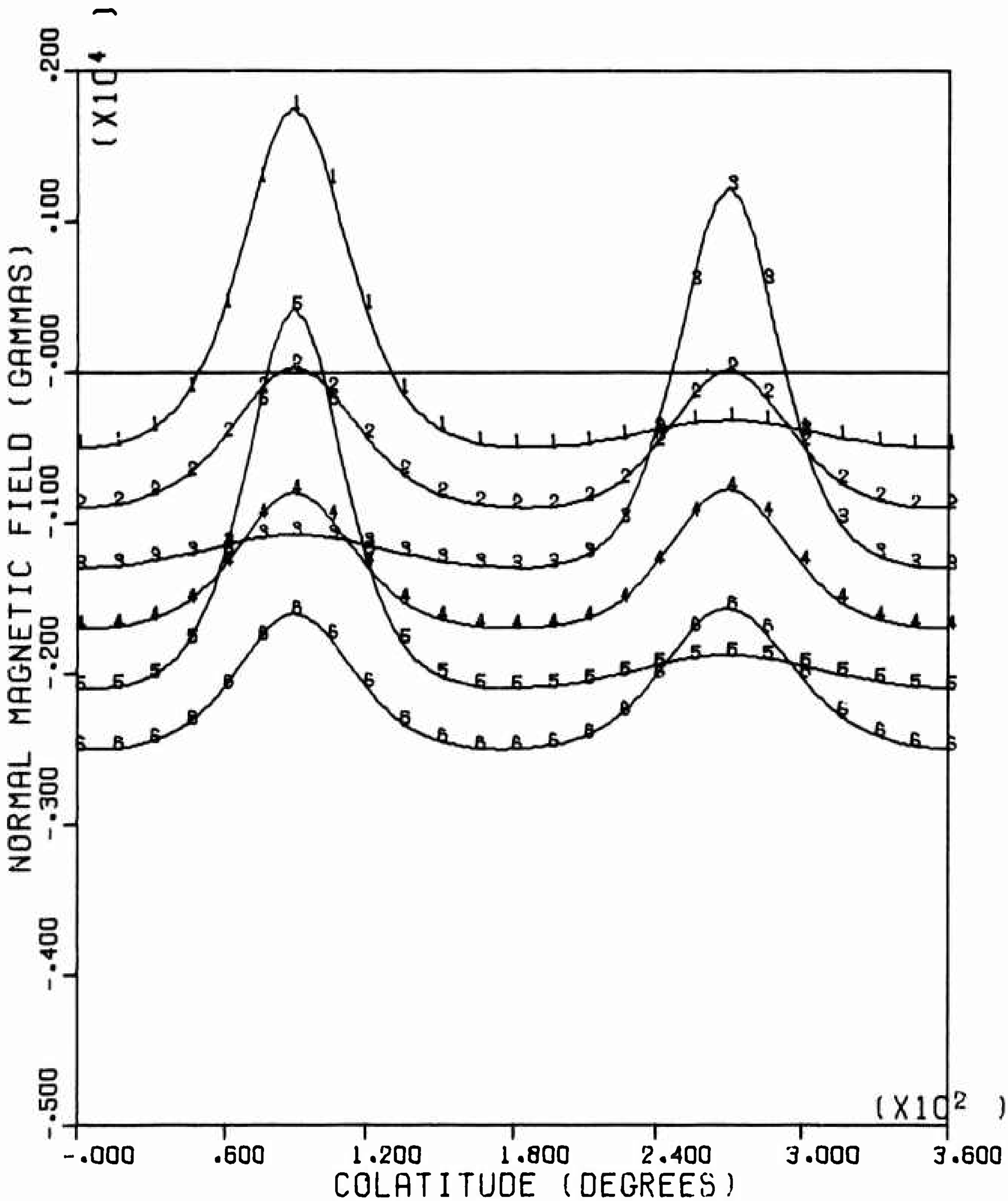


FIG. 6 TYPICAL DATA CURVES

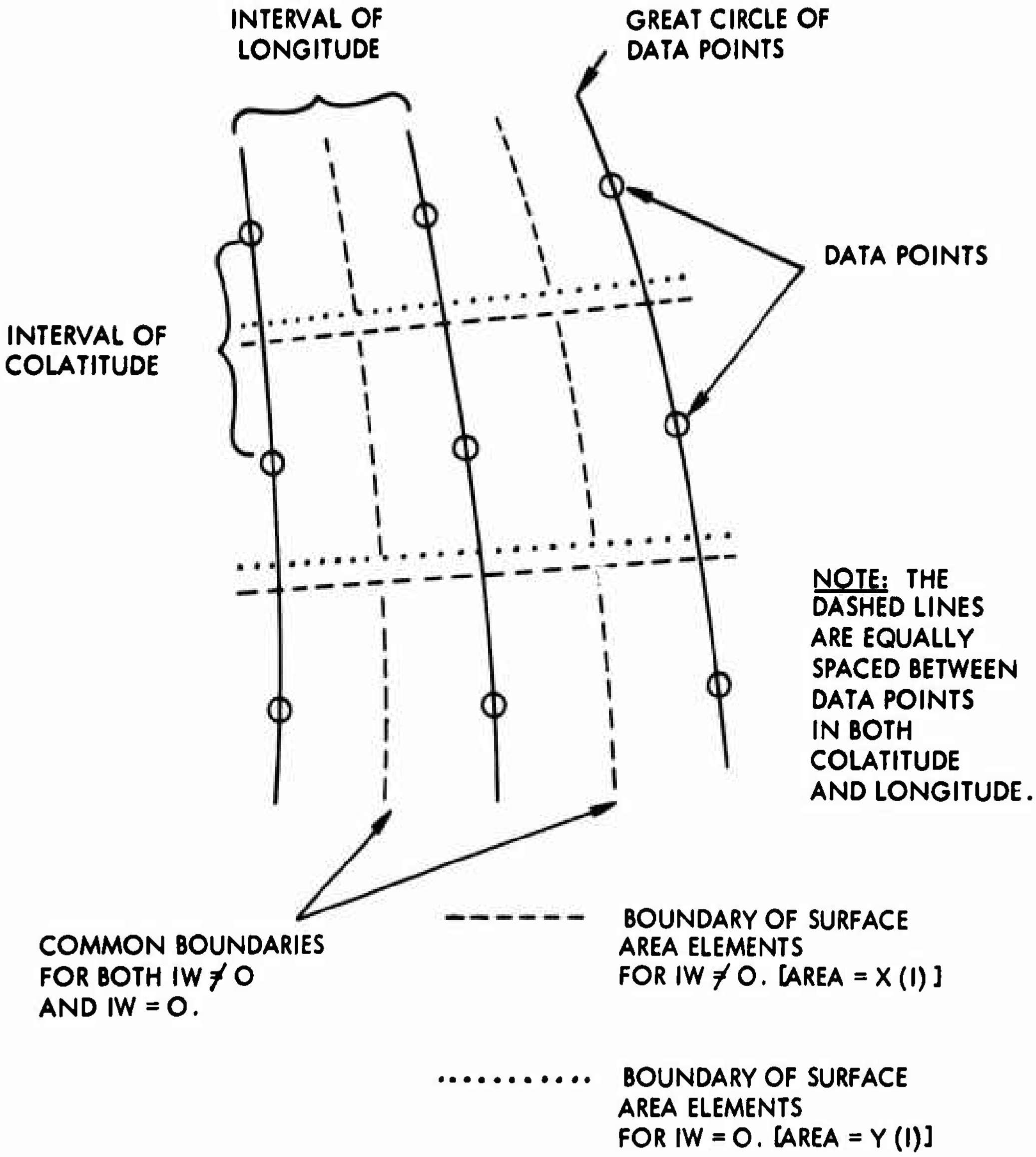


FIG. 7 EXAMPLE OF SURFACE AREA ELEMENTS ASSIGNED TO DATA POINT

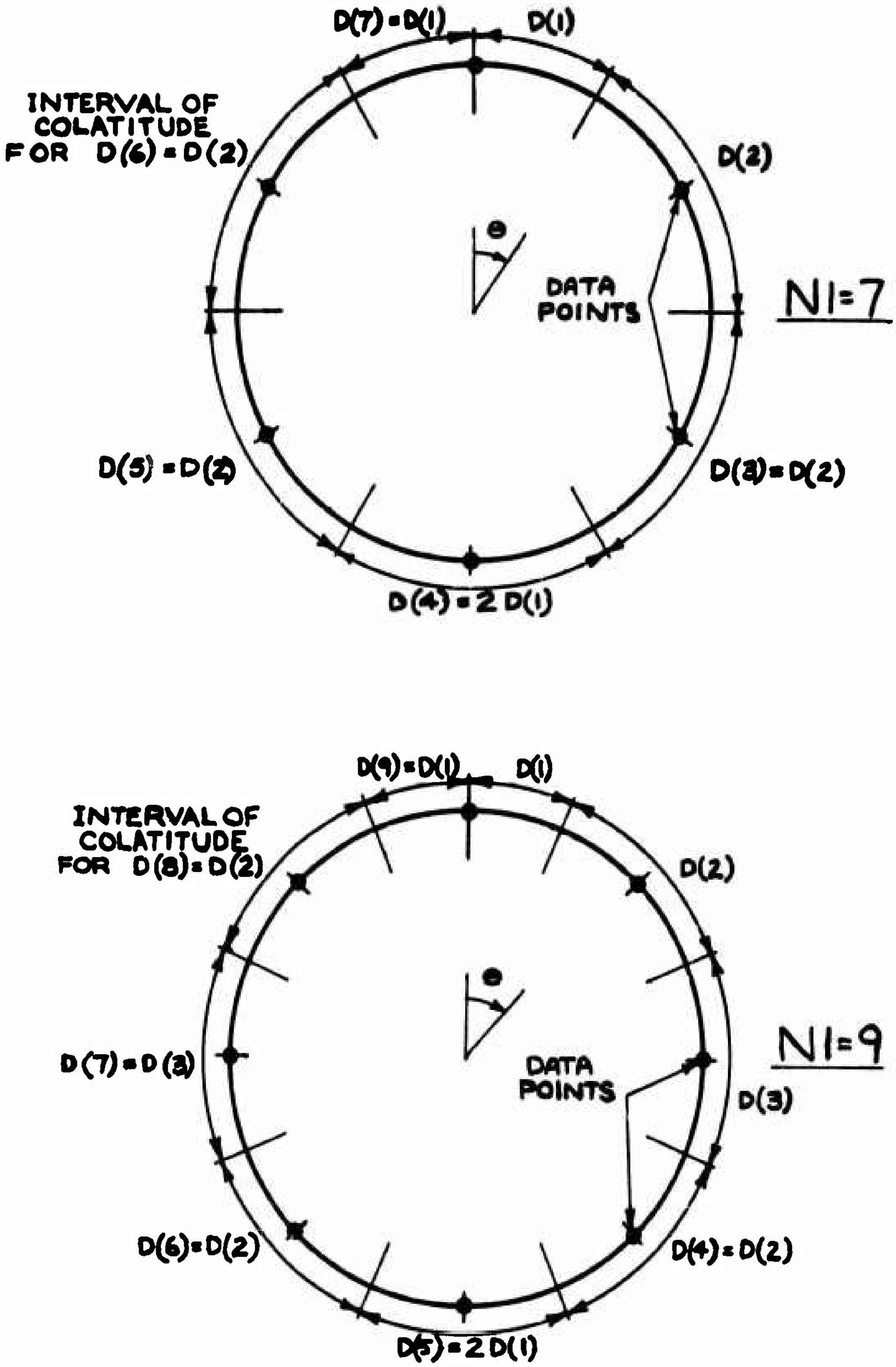


FIG. 8 EXAMPLES OF INTERVALS OF COLATITUDE

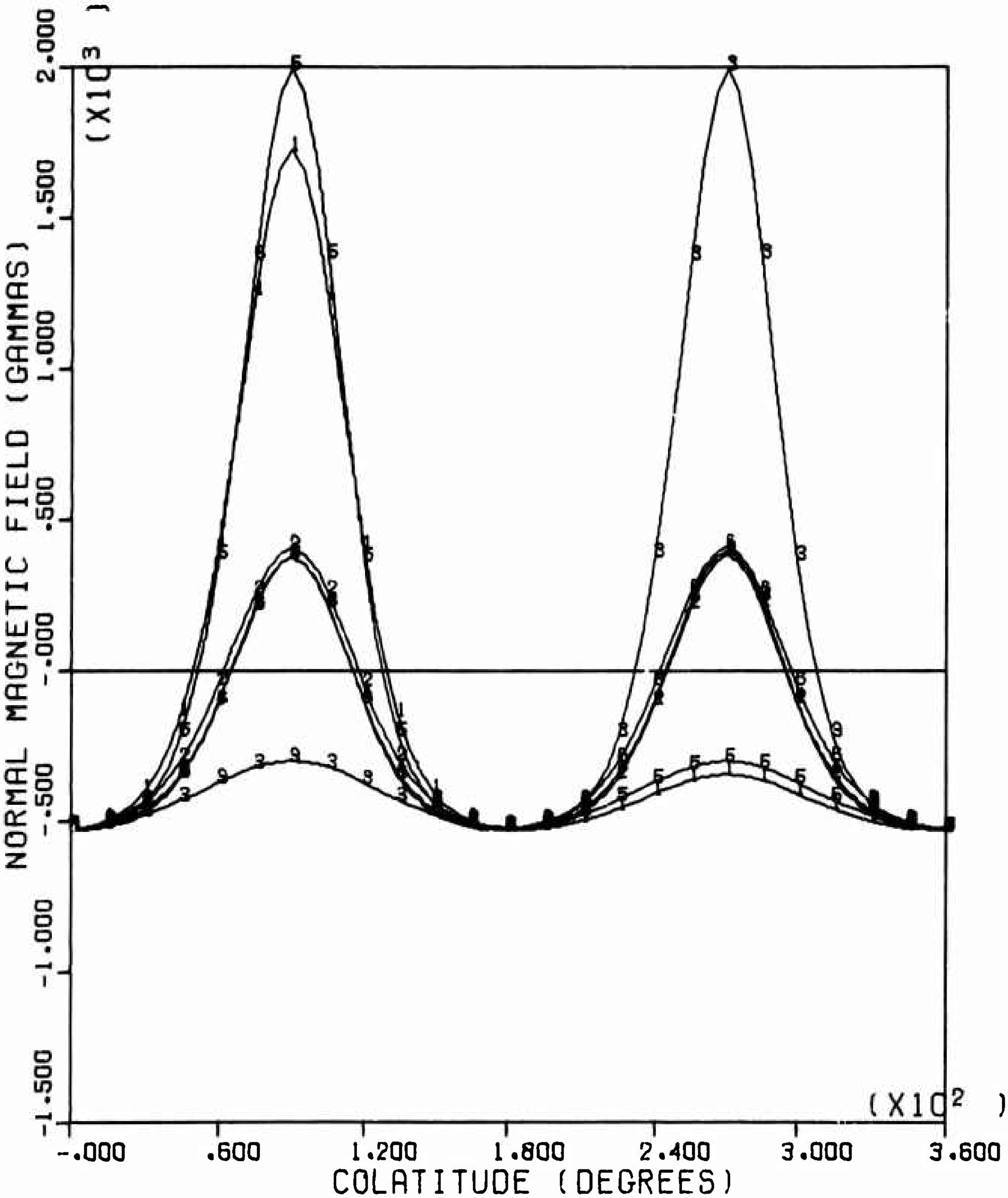


FIG. 9 DATA CURVES FROM SA3024 FOR SAMPLE PROBLEM

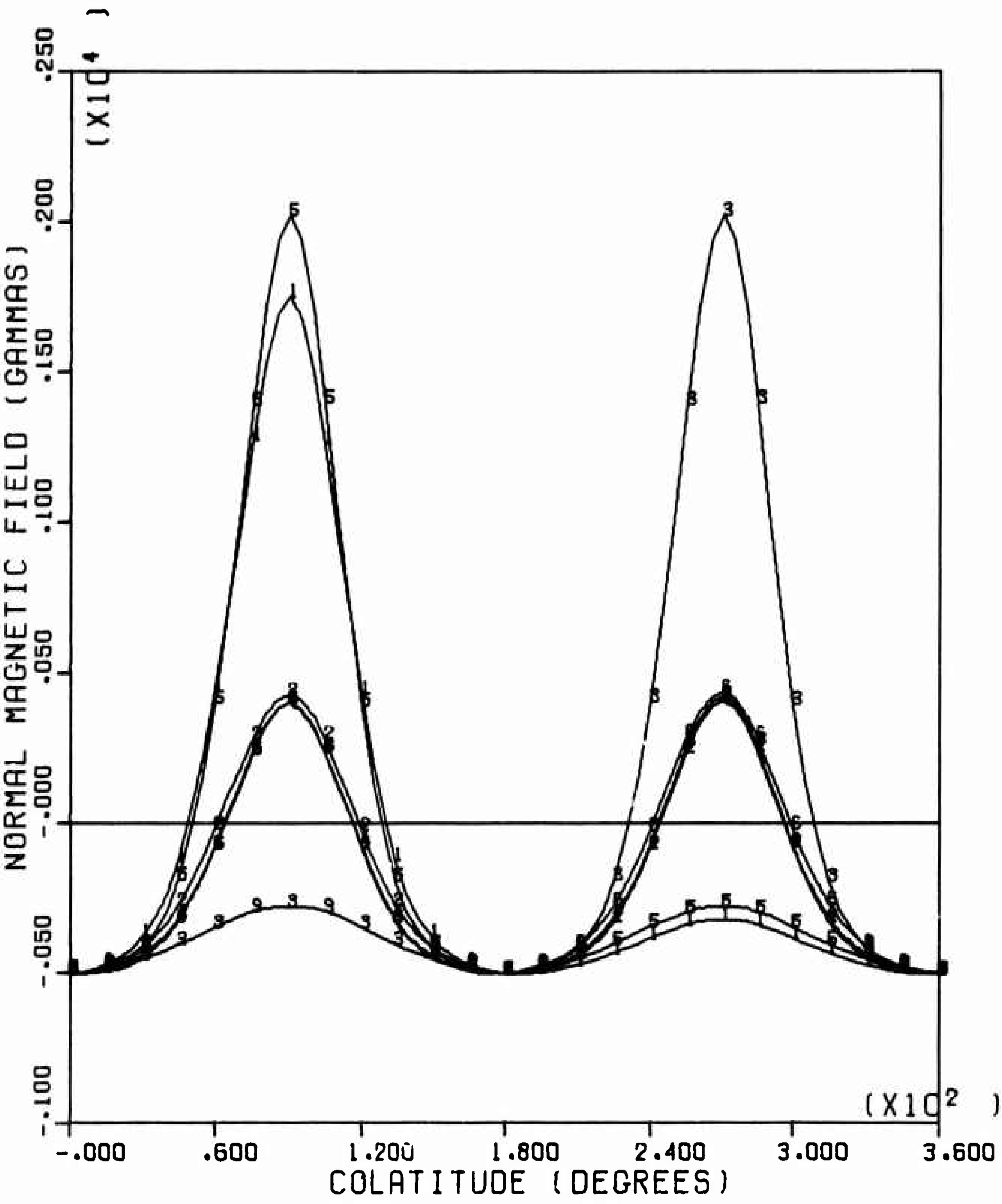


FIG. 10 DATA CURVES FROM SA4024 FOR SAMPLE PROBLEM

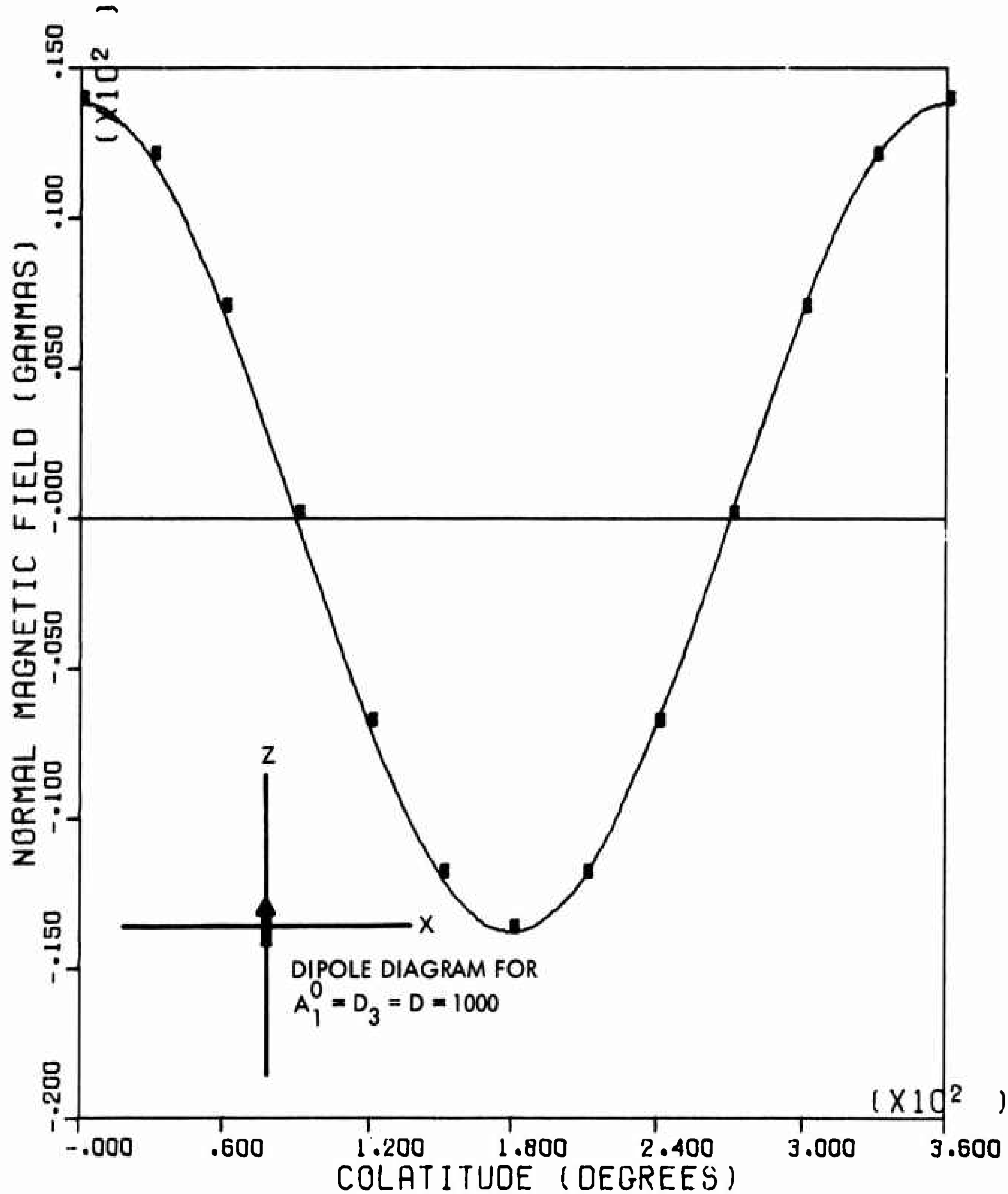


FIG. 11 DIPOLE CURVES FOR $A_1^0 = D_3 = 1000$ GAUSS-CM^3

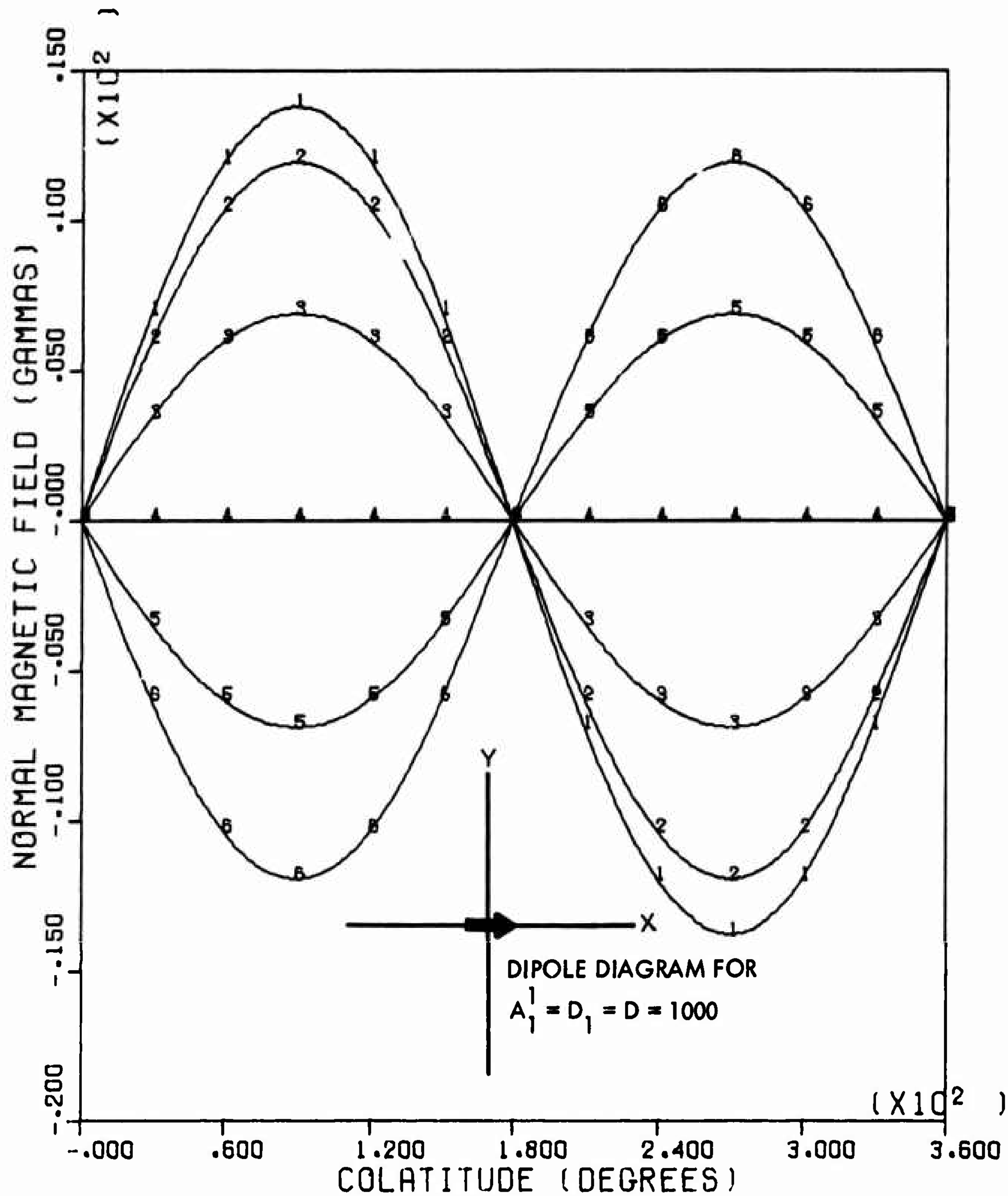


FIG. 12 DIPOLE CURVES FOR $A_1^1 = D_1 = 1000$ GAUSS-CM^3

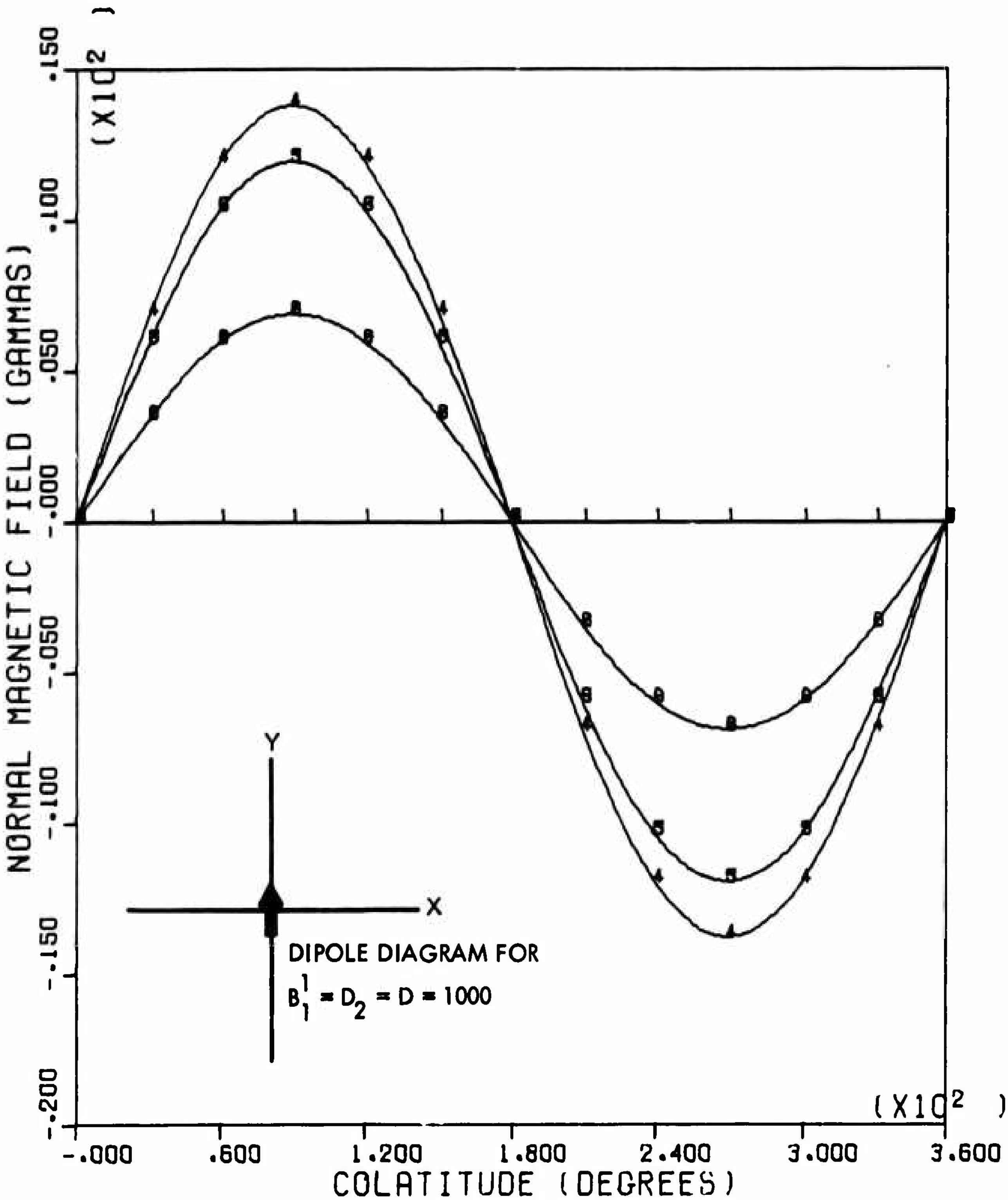


FIG. 13 DIPOLE CURVES FOR $B_1^1 = D_2 = 1000$ GAUSS-CM^3

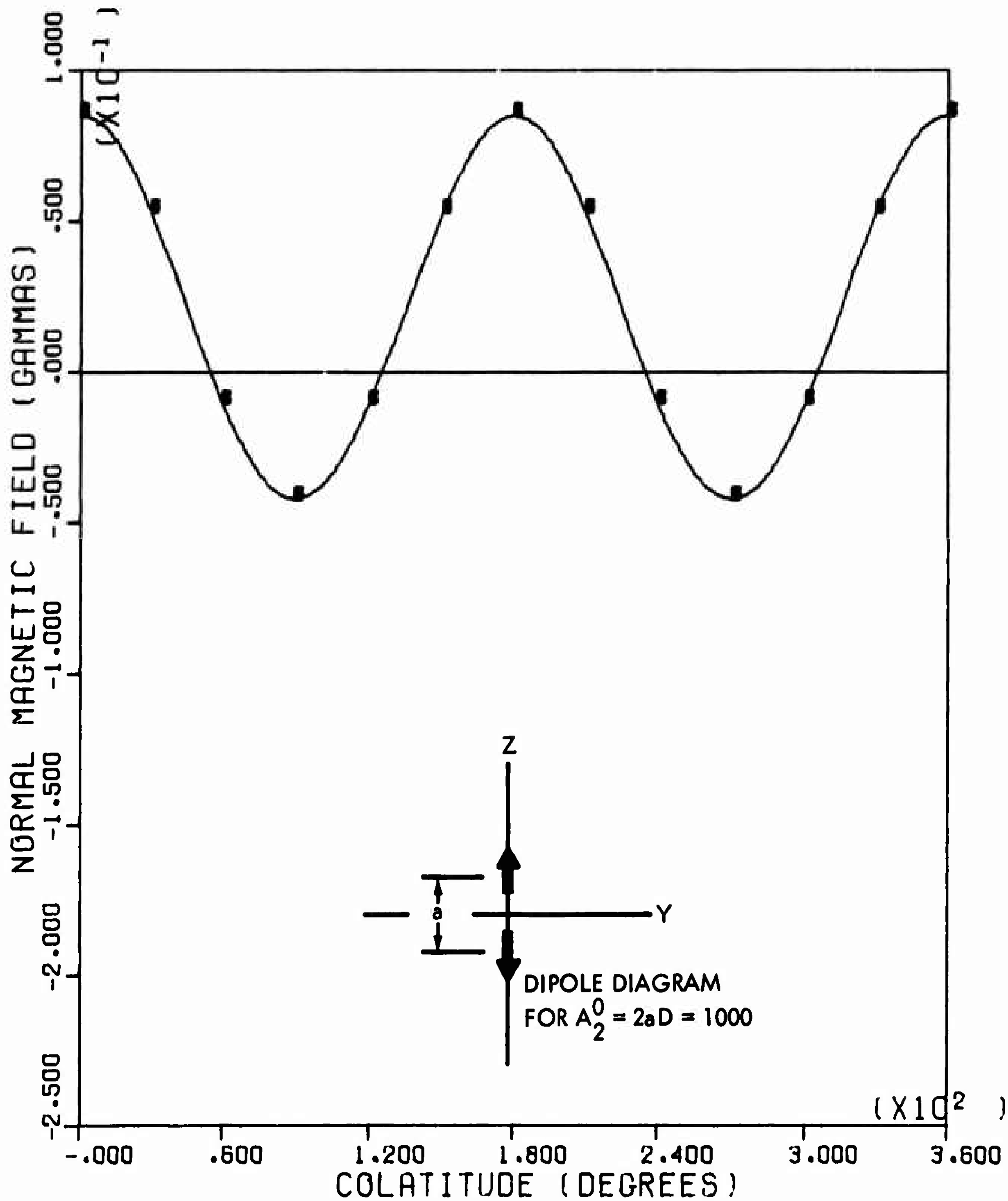


FIG. 14 QUADRUPOLE CURVES FOR $A_2^0 = 1000$ GAUSS-CM^4

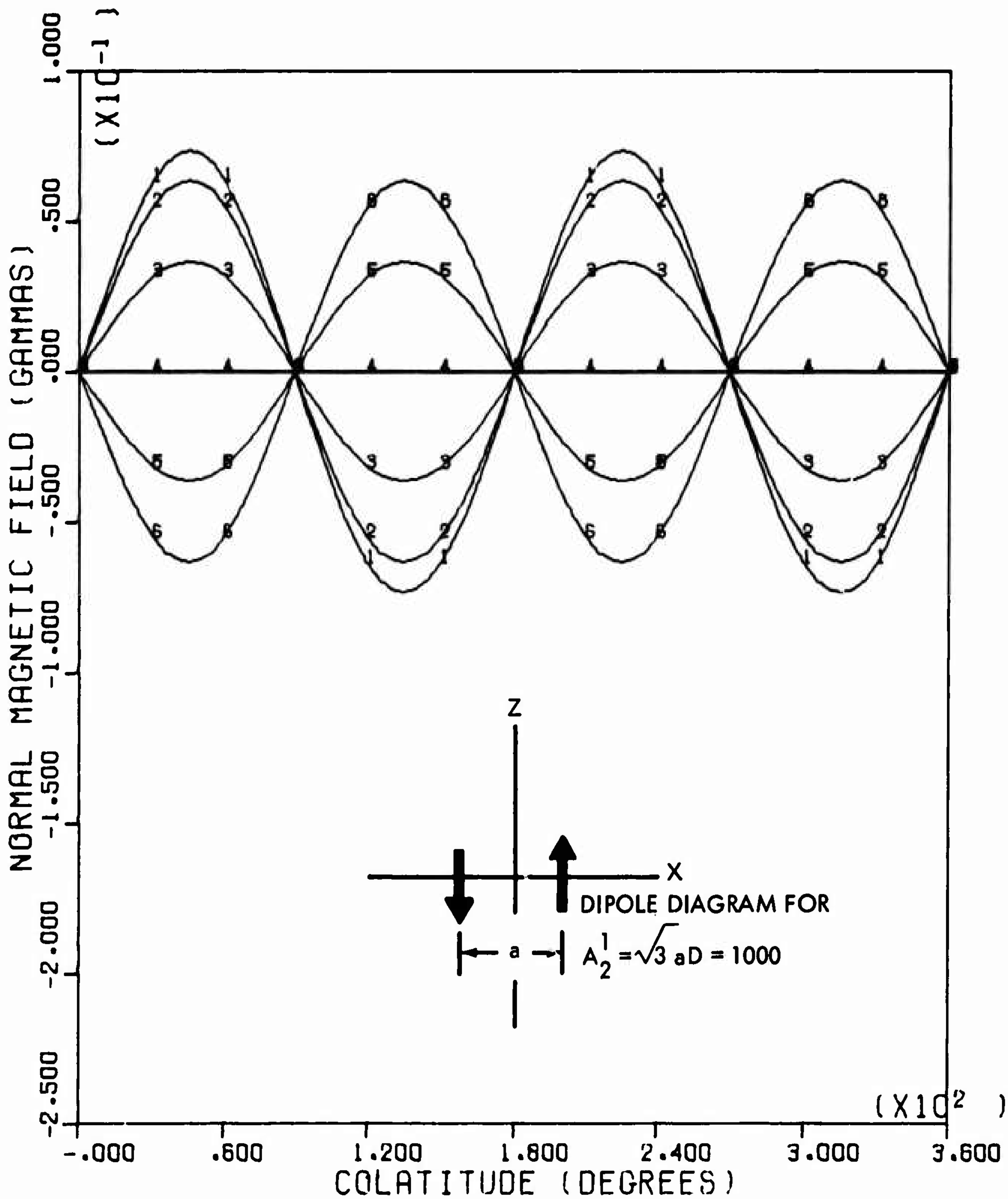


FIG. 15 QUADRUPOLE CURVES FOR A_2^1 = 1000 GAUSS-CM^4

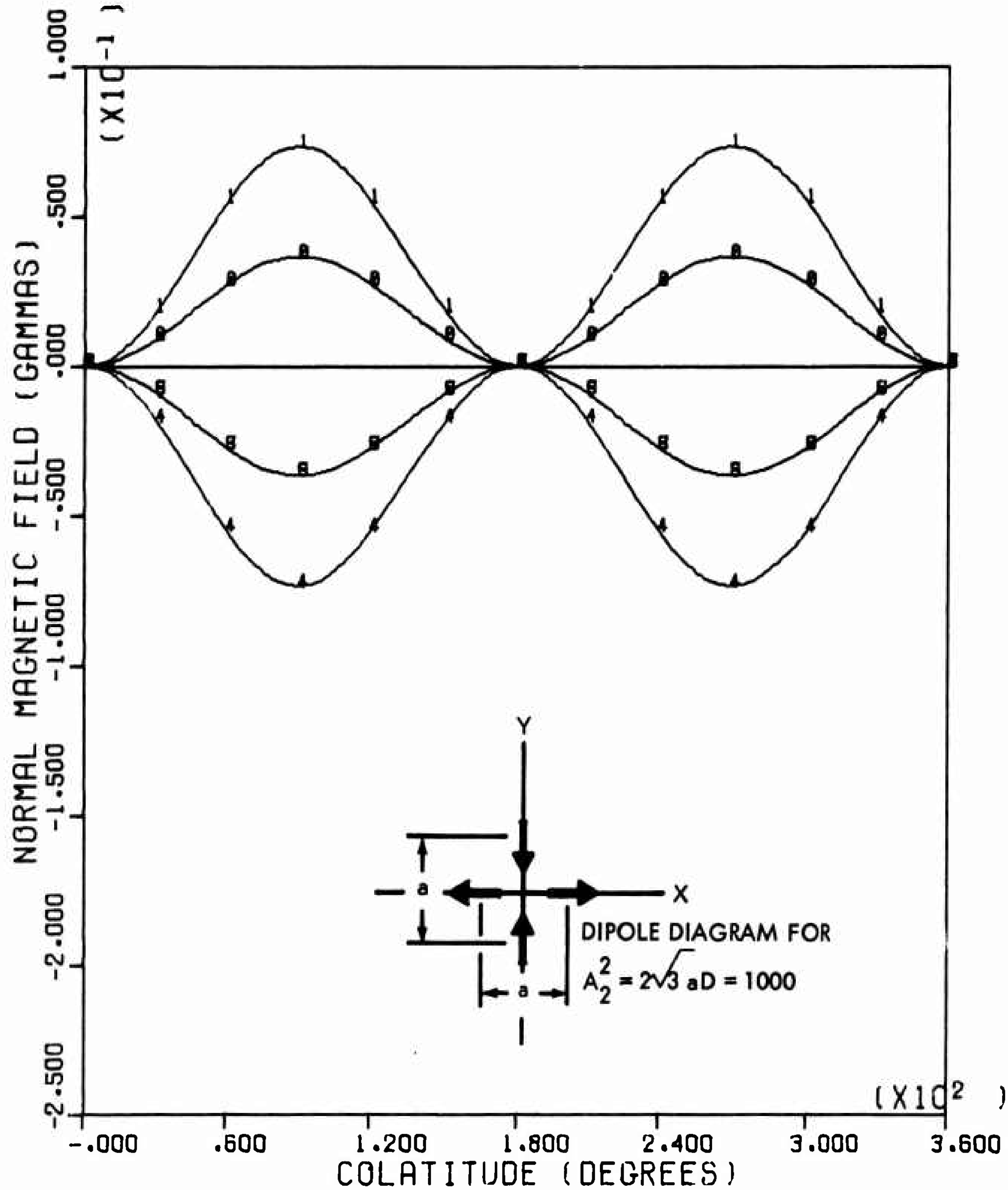


FIG. 16 QUADRUPOLE CURVES FOR $A_2^2 = 1000$ GAUSS-CM^4

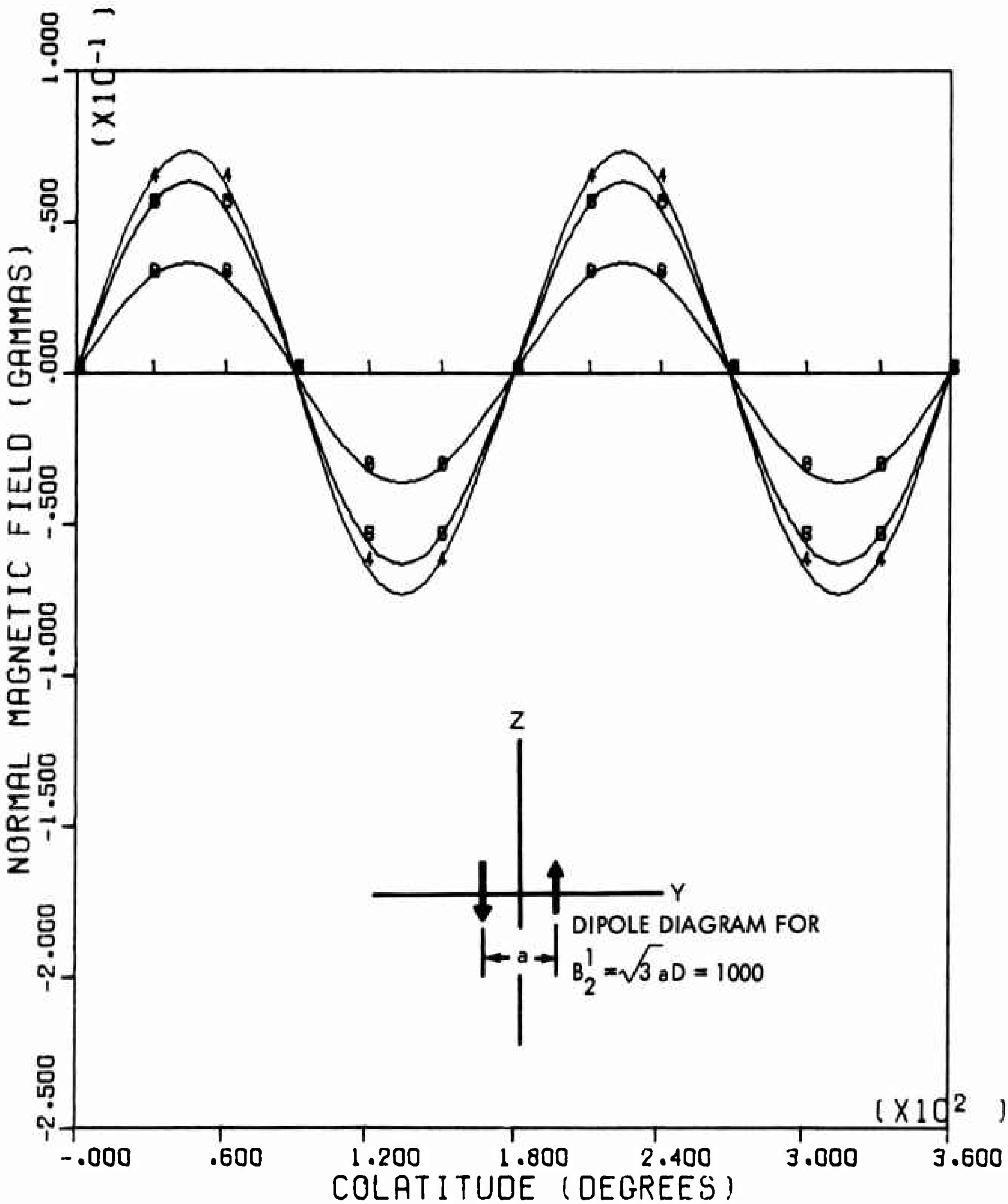


FIG. 17 QUADRUPOLE CURVES FOR $B_2^1 = 1000$ GAUSS-CM^4

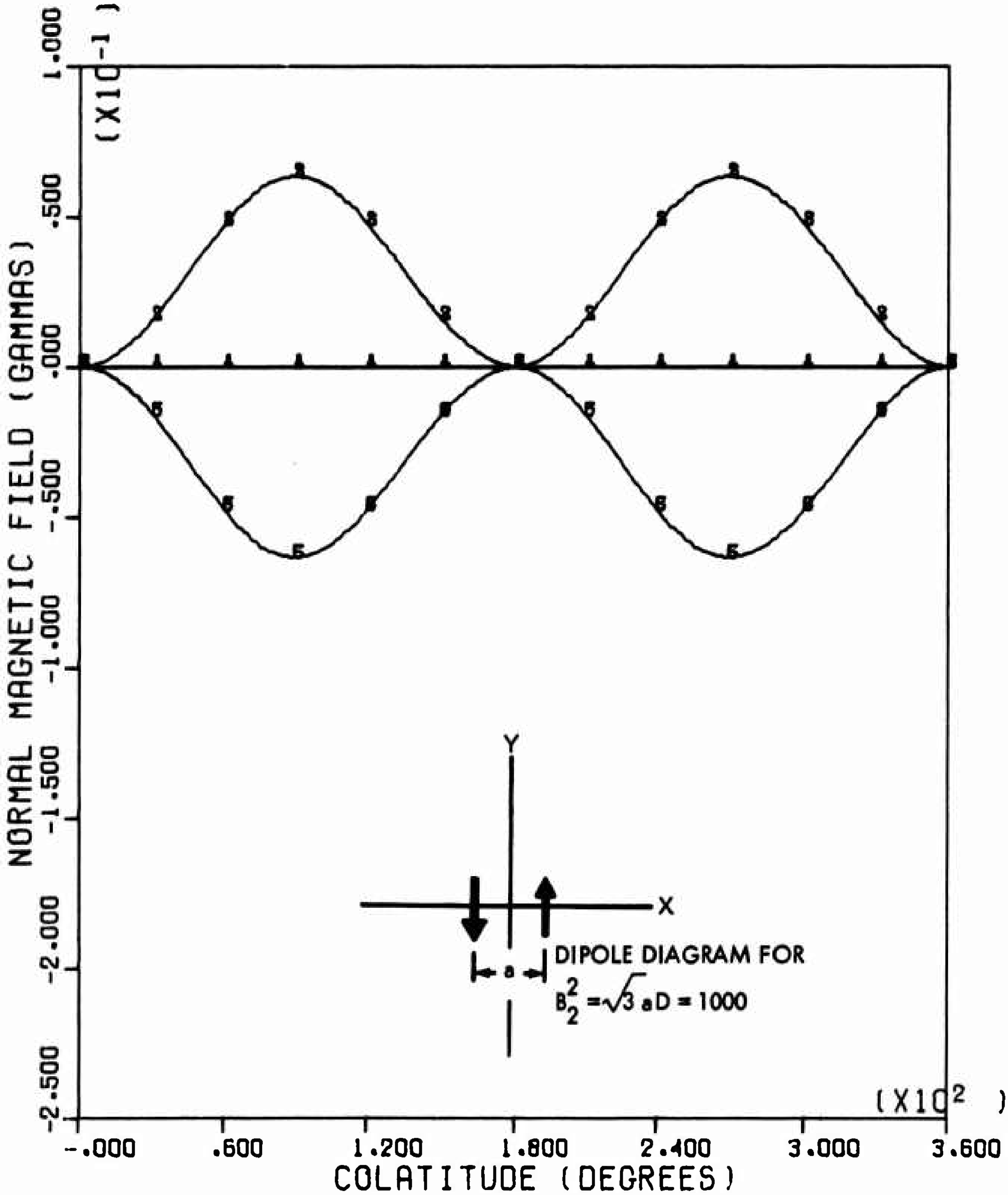


FIG. 18 QUADRUPOLE CURVES FOR B_2^2 = 1000 GAUSS-CM^4

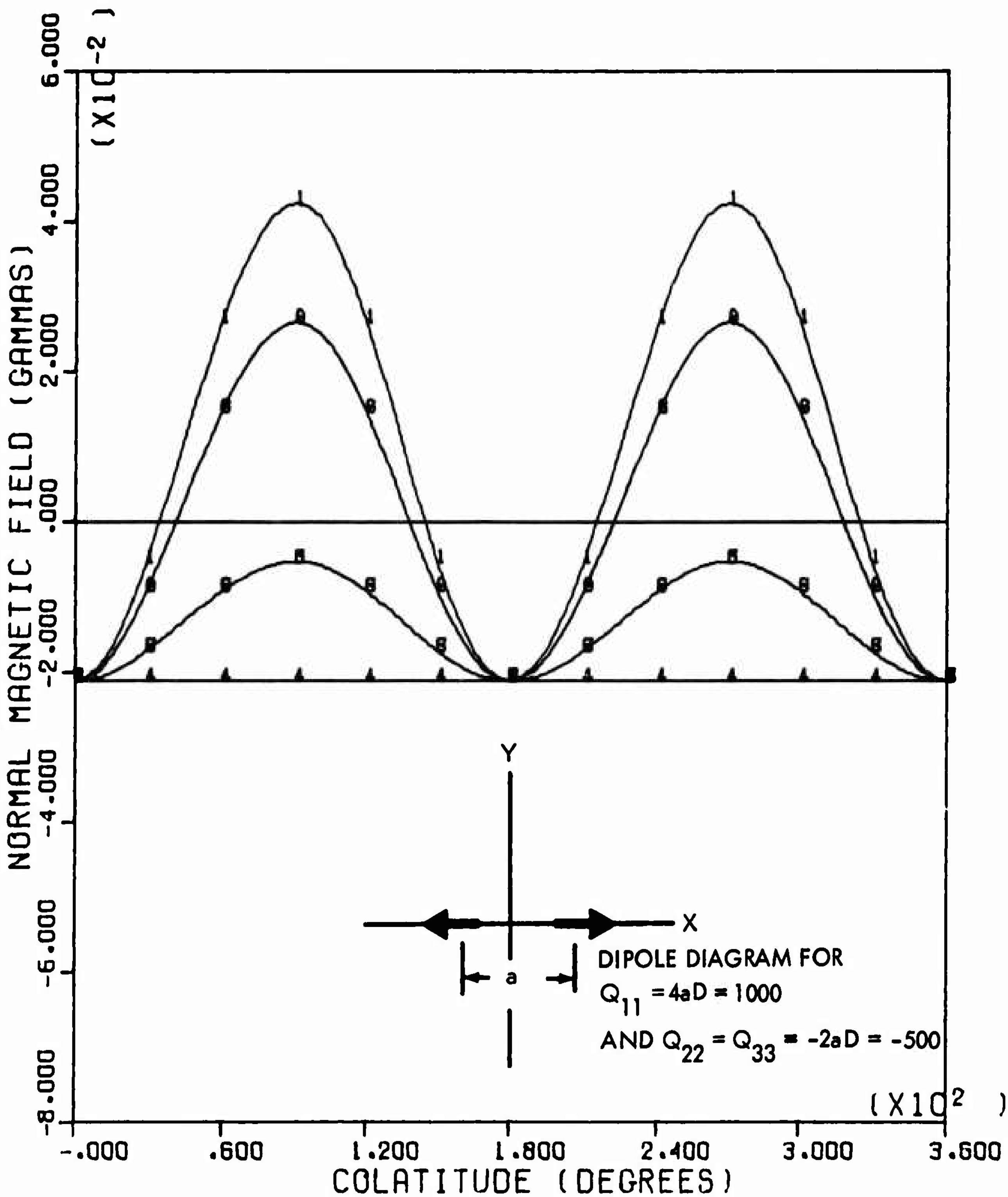


FIG. 19 QUADRUPOLE CURVES FOR $Q_{11} = 1000$ GAUSS-CM^4 AND $Q_{22} = Q_{33} = -500$ GAUSS-CM^4

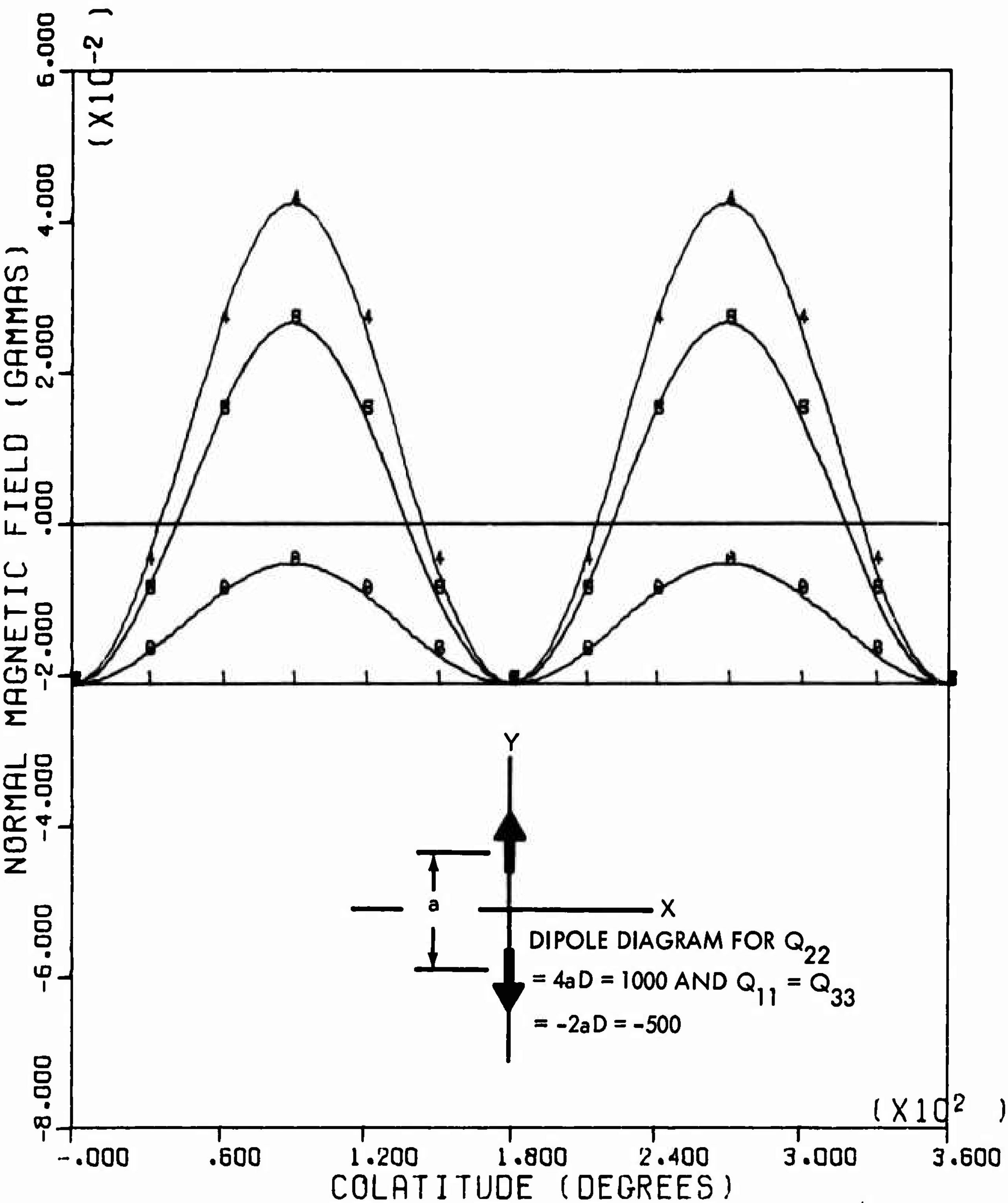


FIG. 20 QUADRUPOLE CURVES FOR Q_{22} = 1000 GAUSS-CM^4 AND $Q_{11} = Q_{33}$ = -500 GAUSS-CM^4

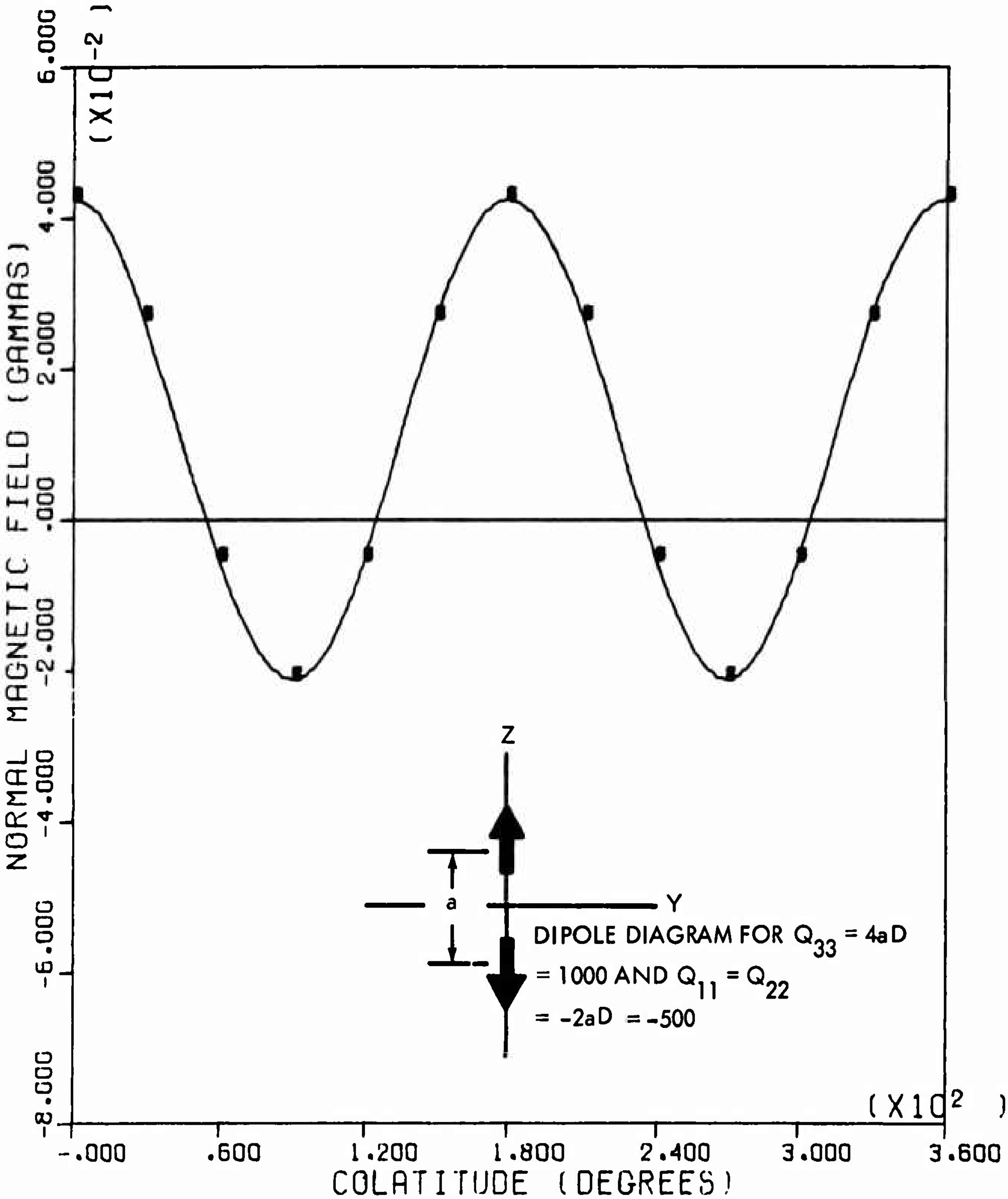
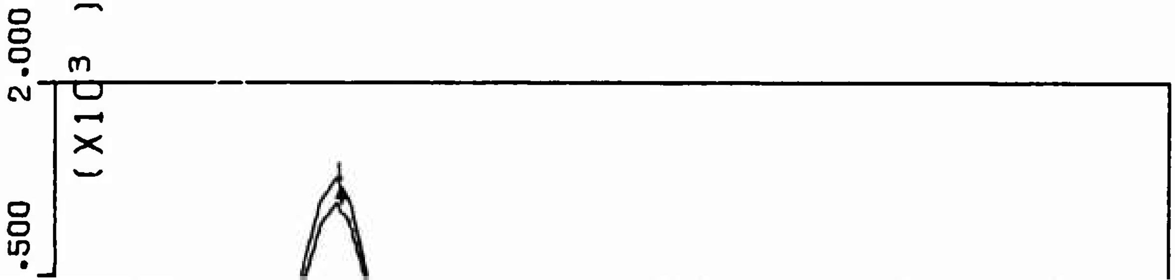


FIG. 21 QUADRUPOLE CURVES FOR Q_{33} = 1000 GAUSS-CM^4
AND $Q_{11} = Q_{22}$ = -500 GAUSS-CM^4


2.000
(X10^3)
.500

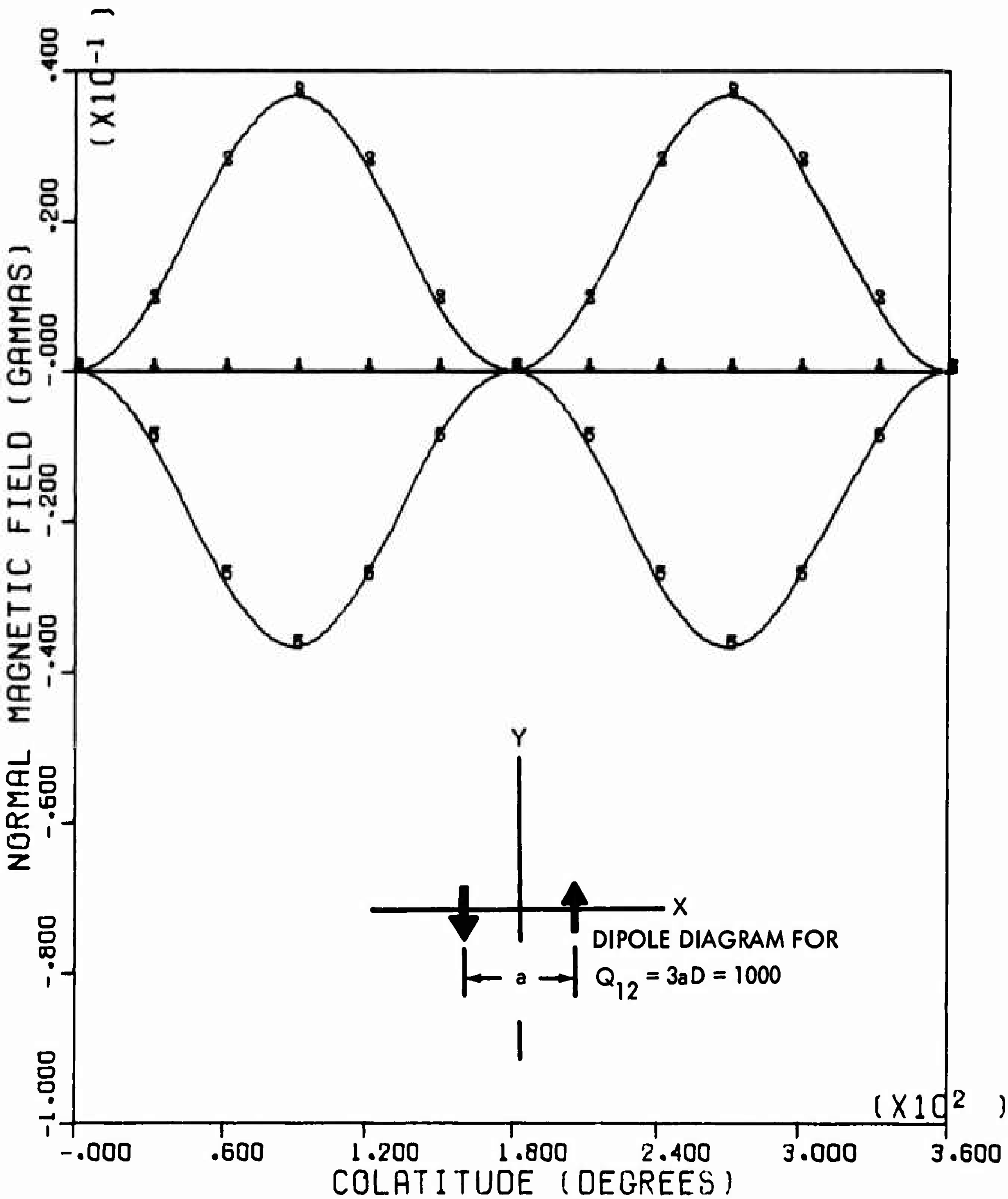
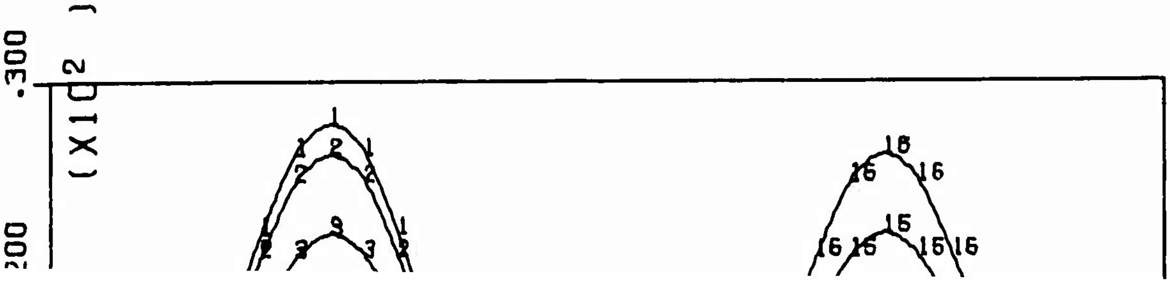


FIG. 22 QUADRUPOLE CURVES FOR Q_{12} = 1000 GAUSS-CM^4


.300
(X10^2)
.200

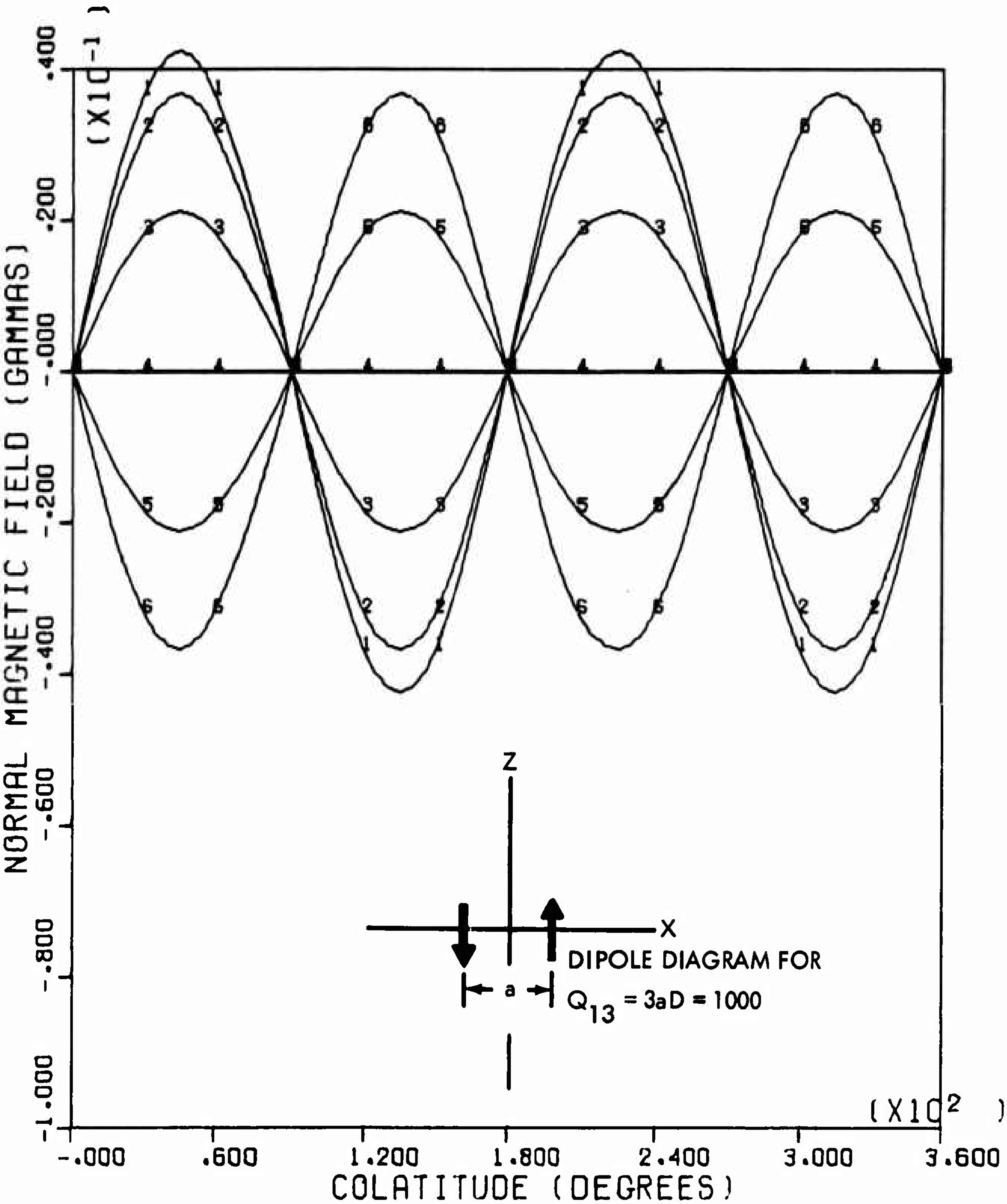
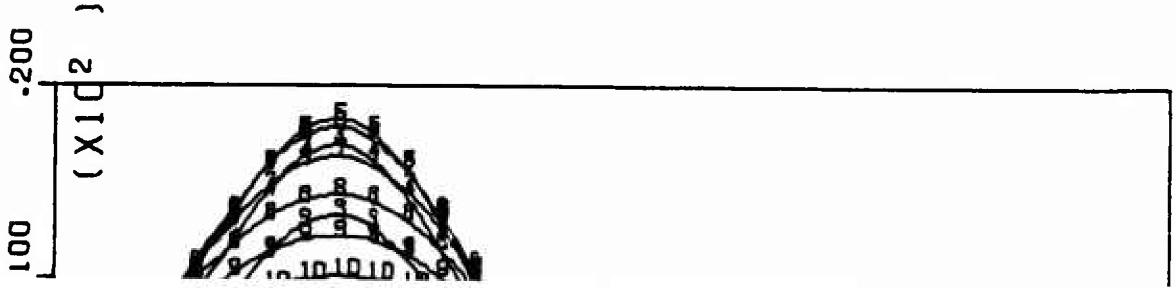


FIG. 23 QUADRUPOLE CURVES FOR Q_{13} = 1000 GAUSS-CM^4


.200
100

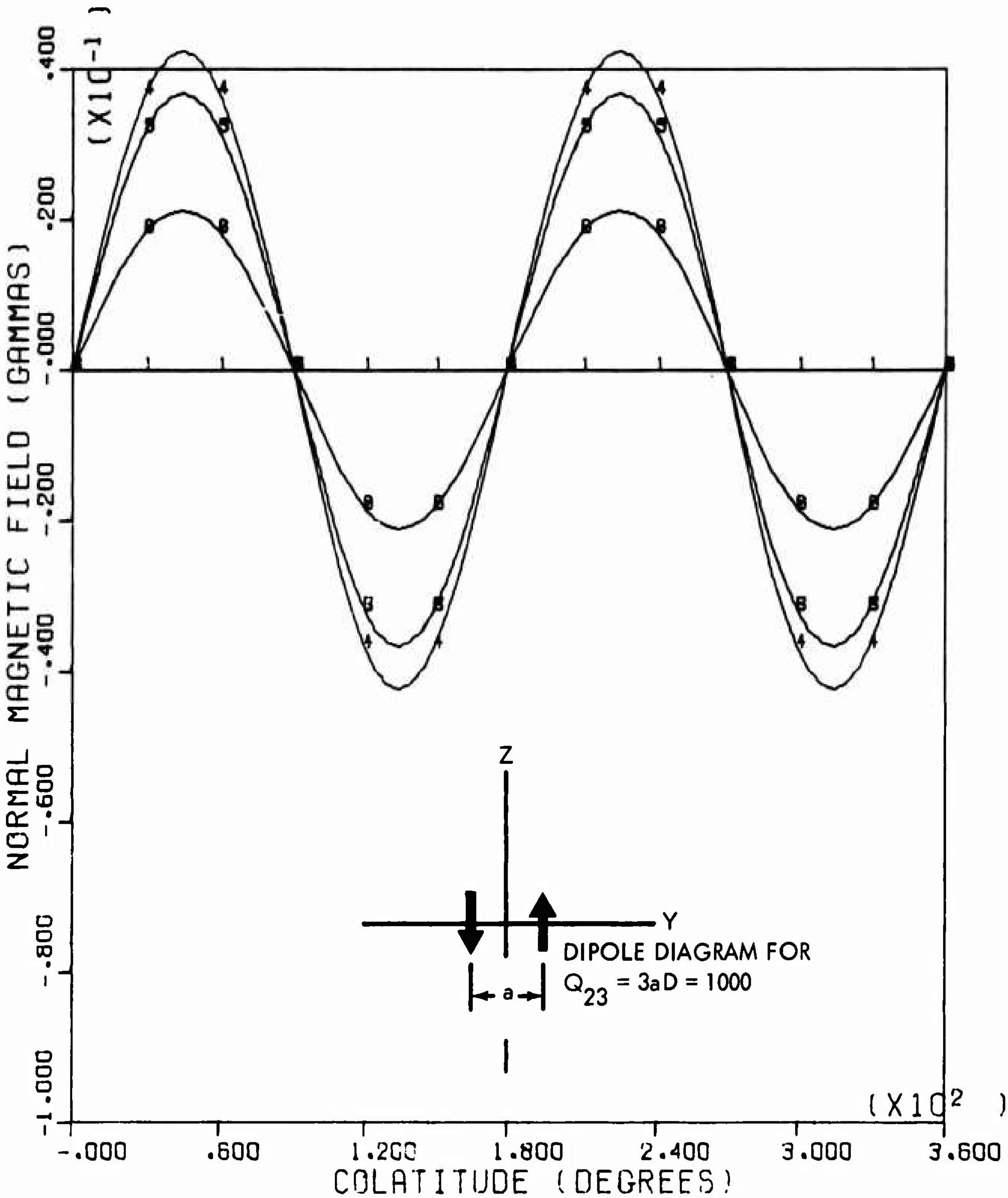


FIG. 24 QUADRUPOLE CURVES FOR Q_{23} = 1000 GAUSS-CM^4

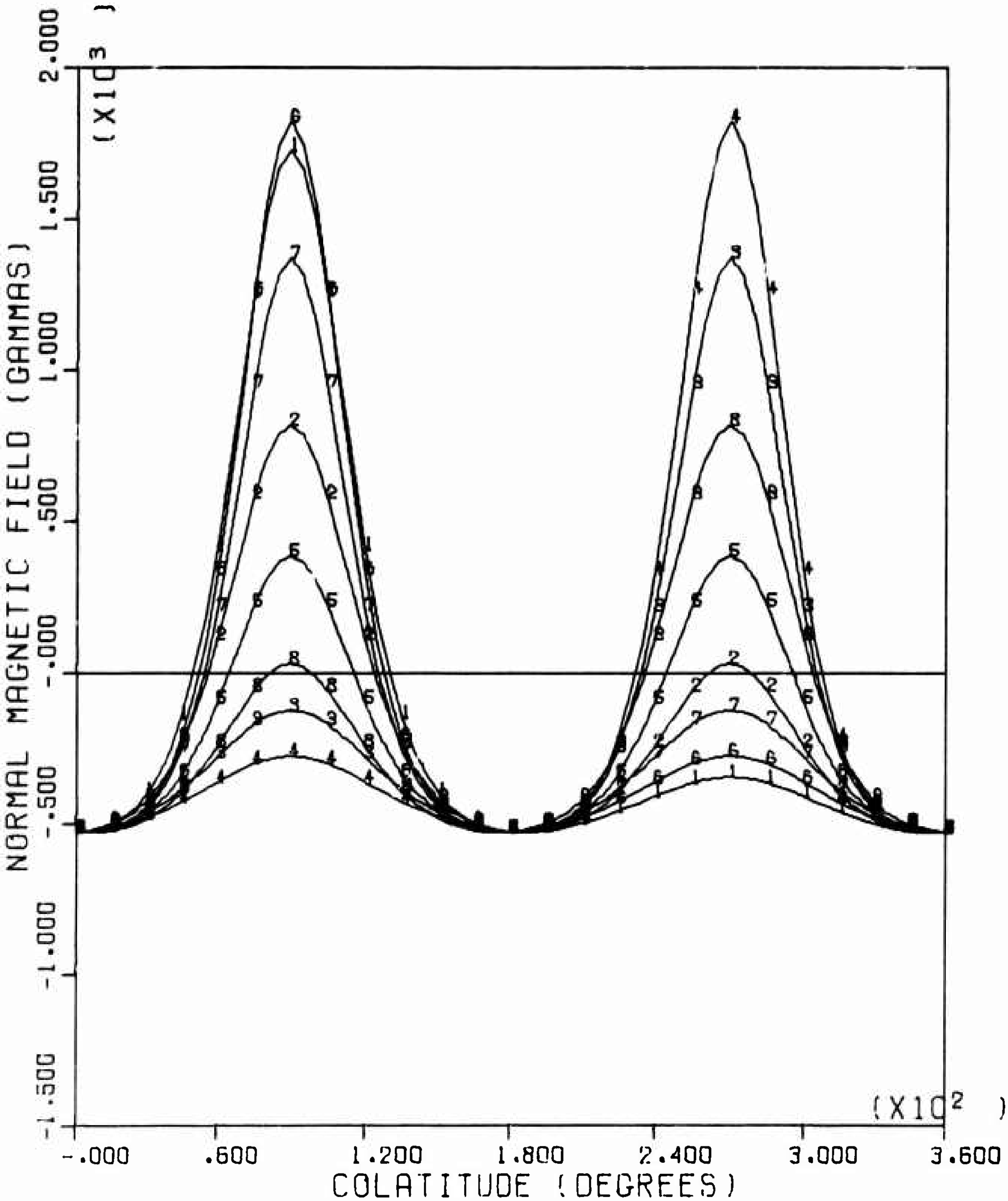


FIG. 25 DATA CURVES FROM SA5024 FOR SAMPLE PROBLEM

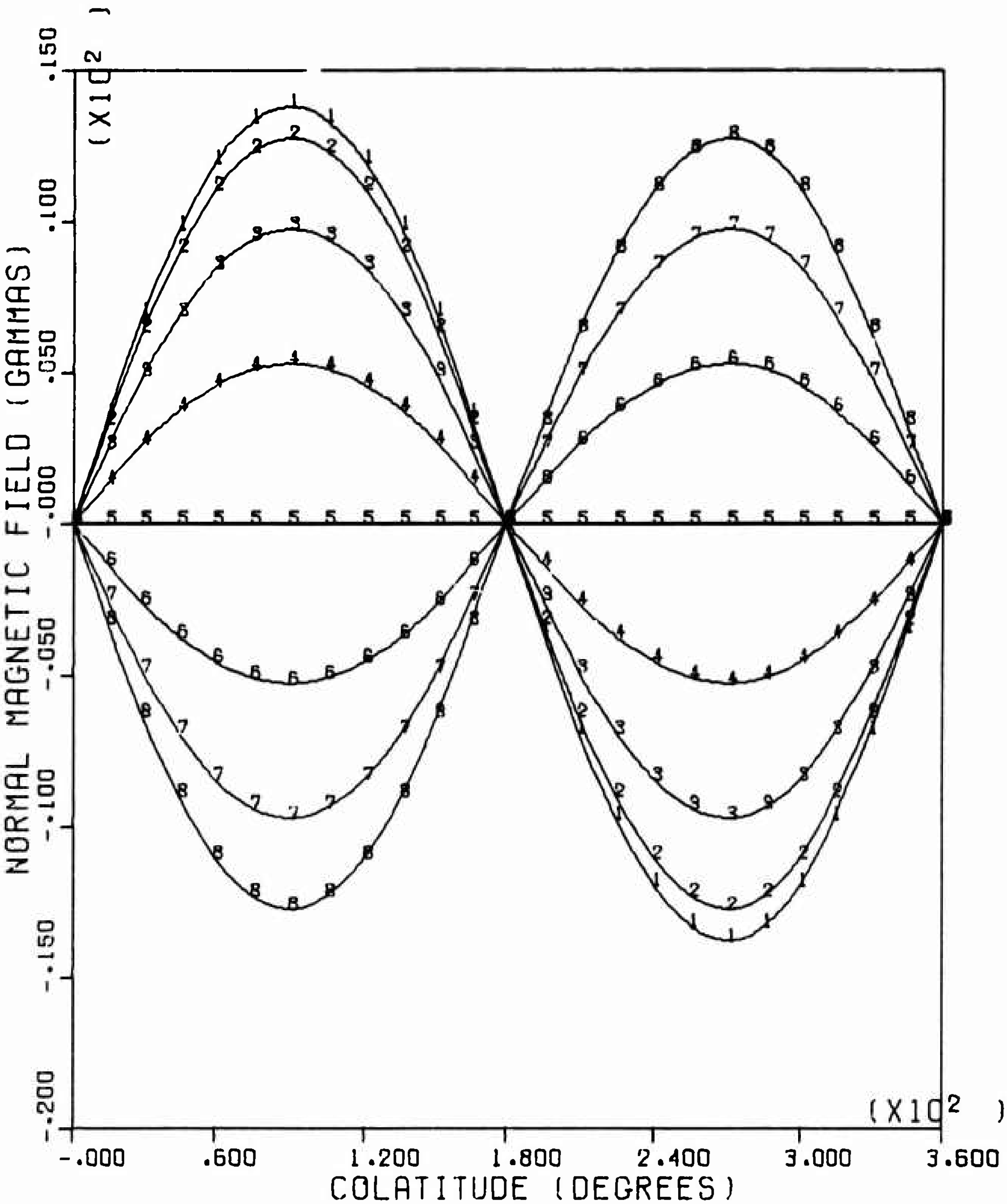


FIG. 26 MULTIPOLE COMPONENT OF DEGREE 1 (DIPOLE COMPONENT)

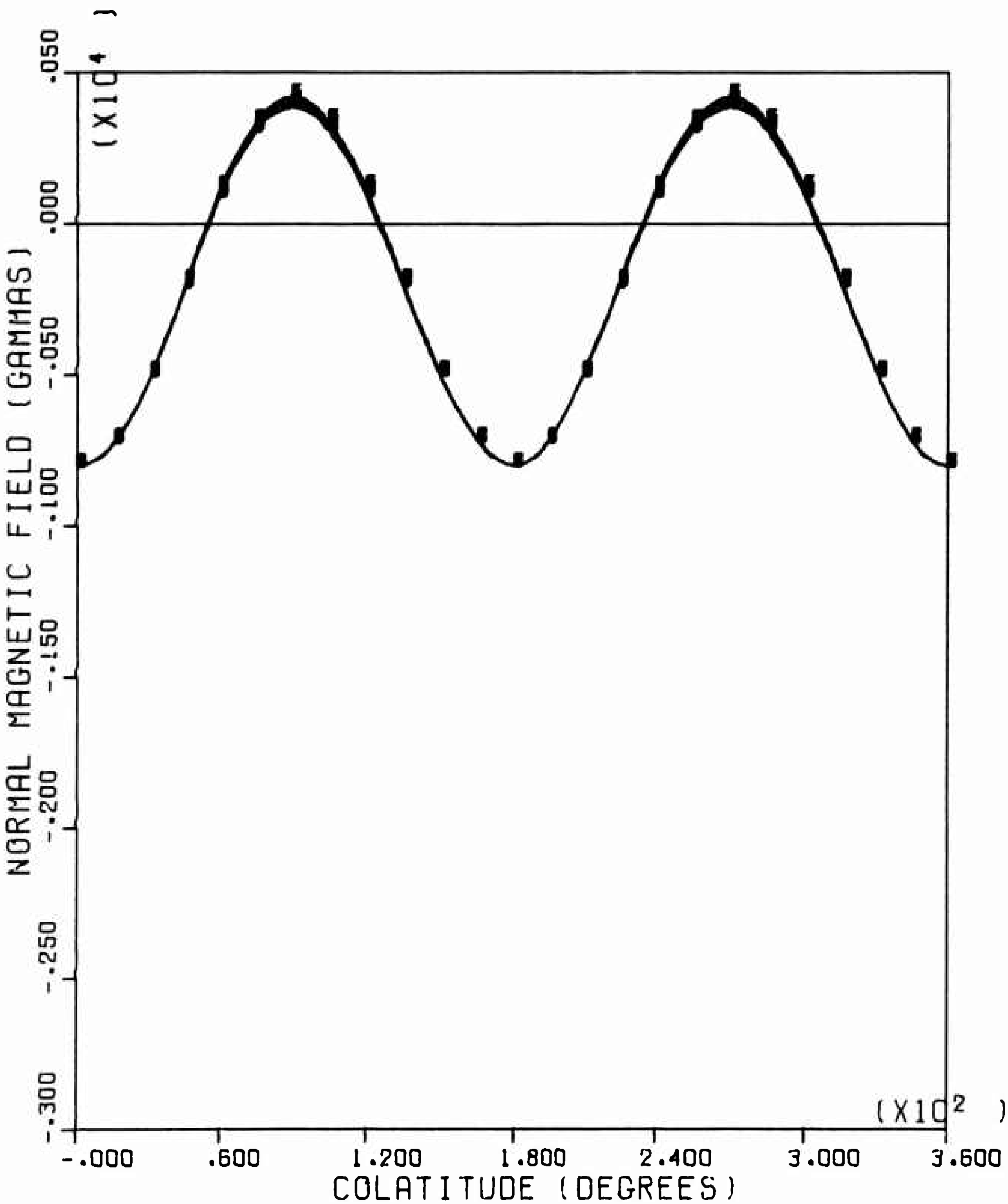


FIG. 27 MULTIPOLE COMPONENT OF DEGREE 2 (QUADRUPOLE COMPONENT)

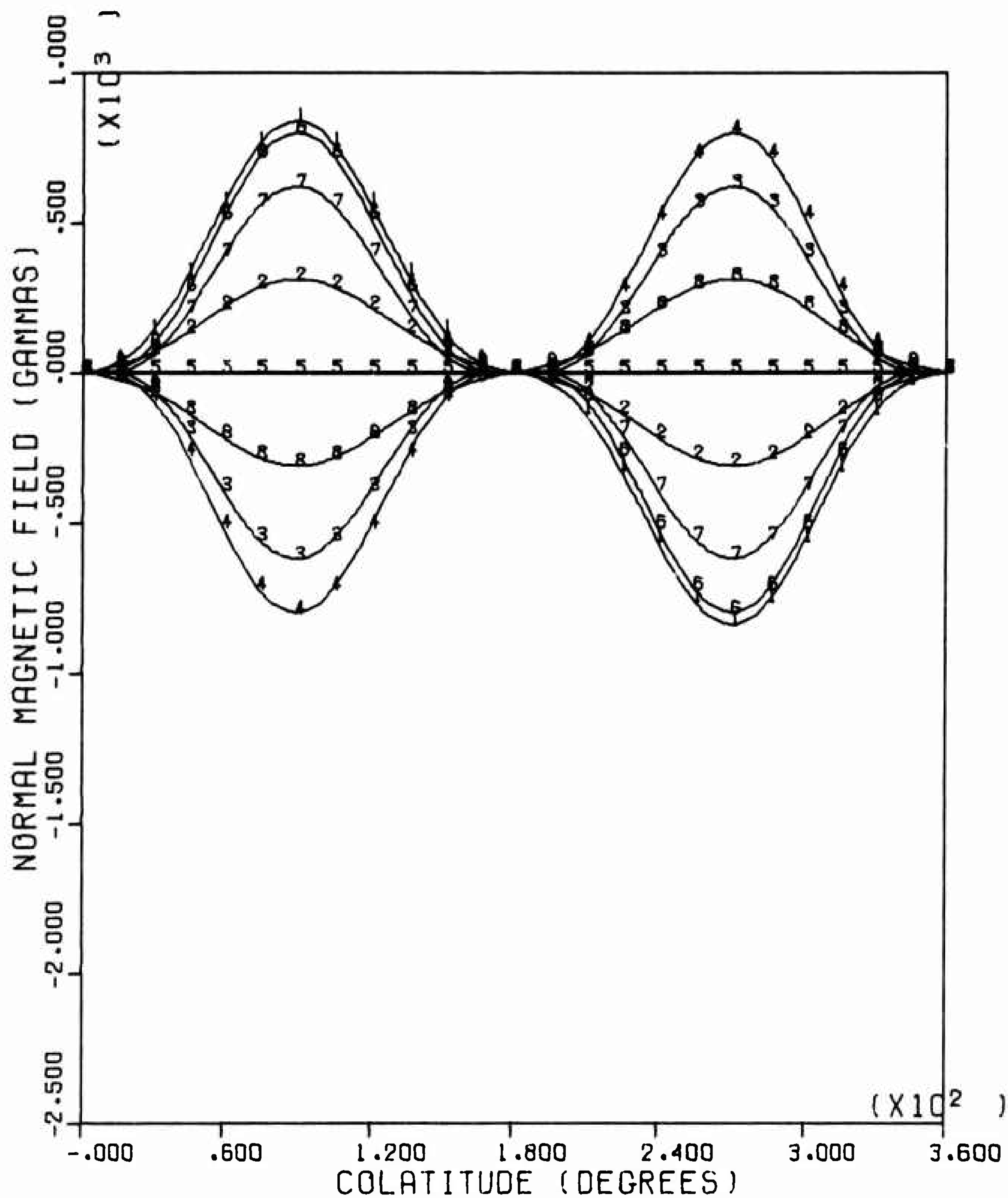


FIG. 28 MULTIPOLE COMPONENT OF DEGREE 3

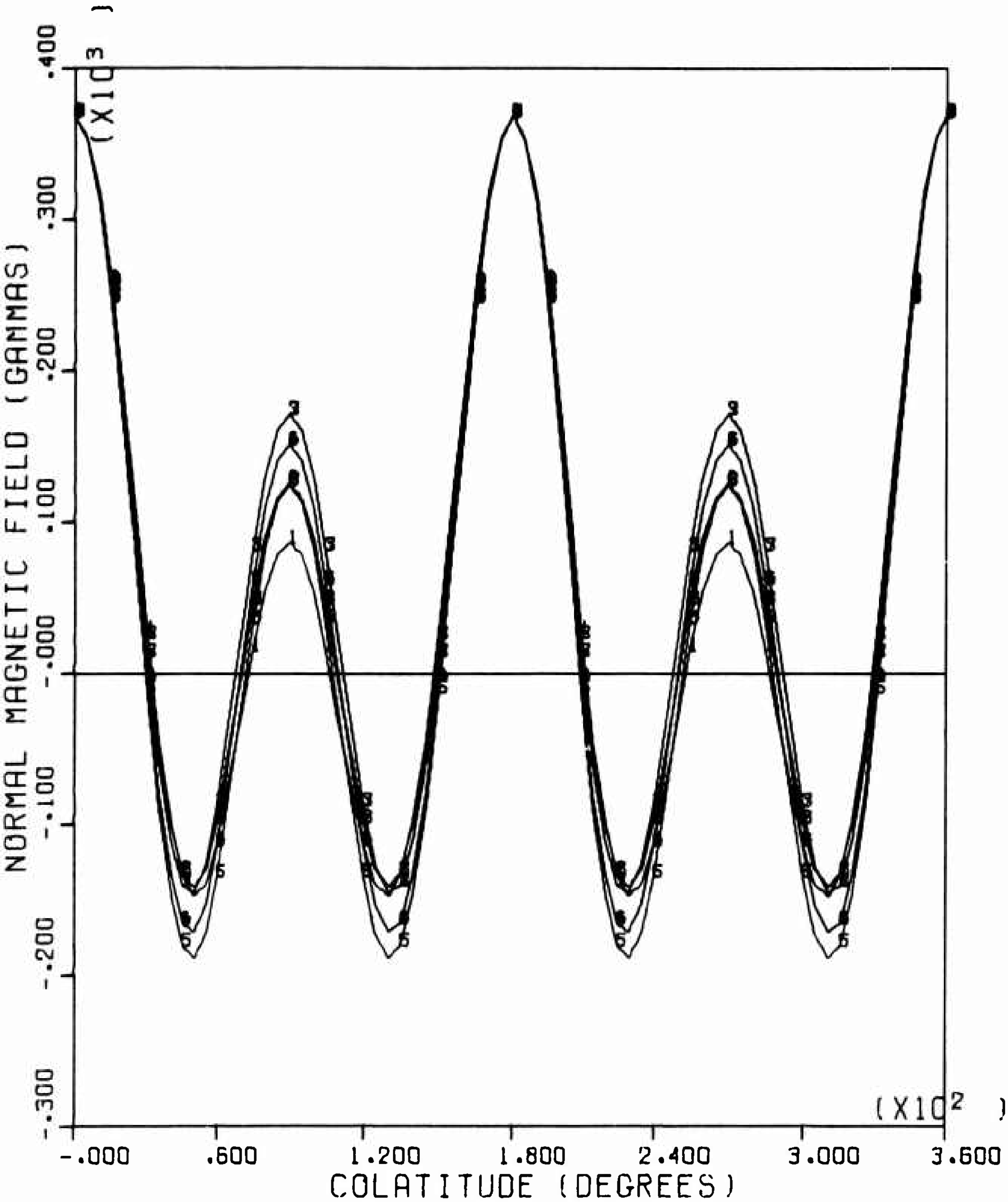


FIG. 29 MULTIPOLE COMPONENT OF DEGREE 4

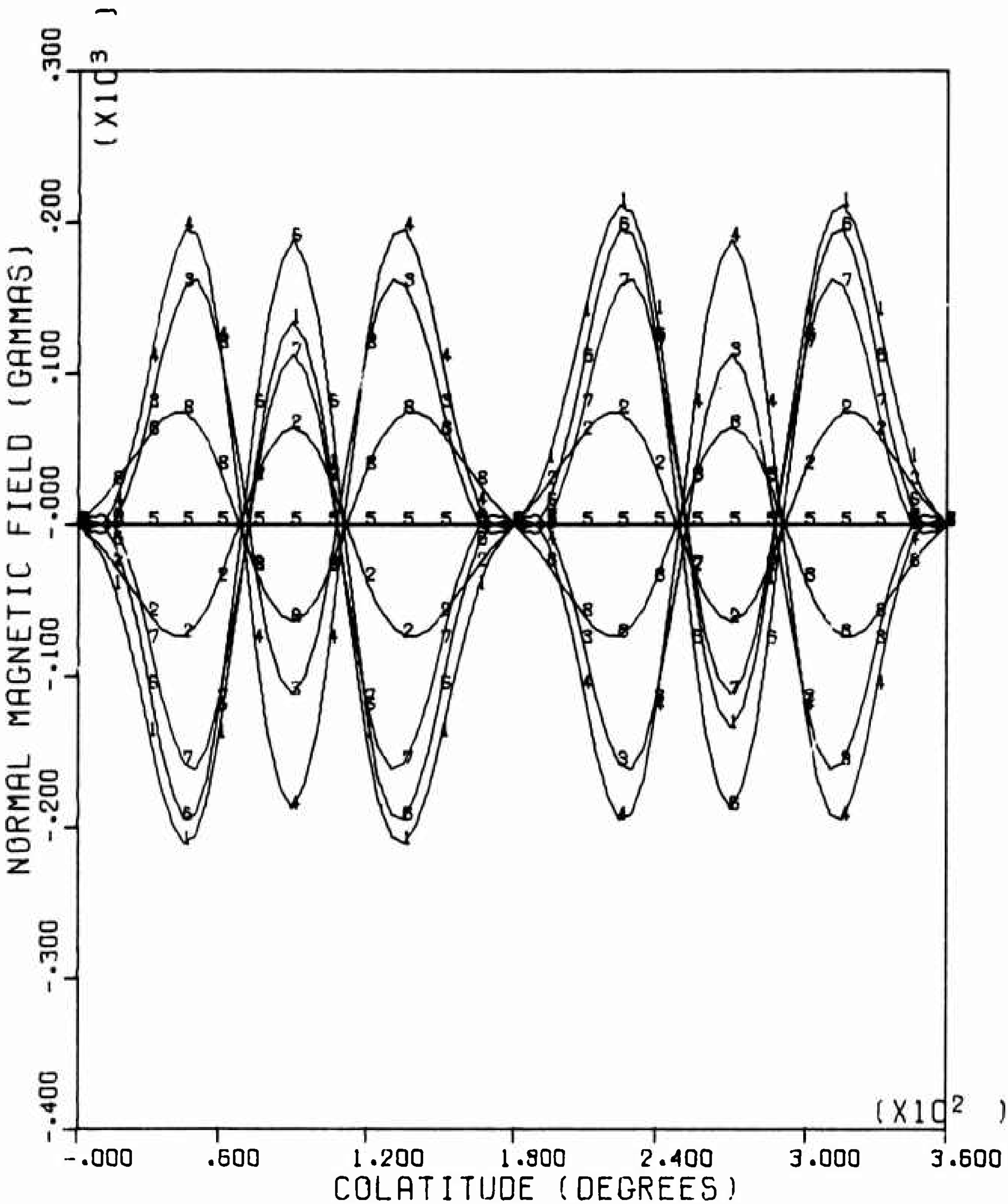


FIG. 30 MULTIPOLE COMPONENT OF DEGREE 5

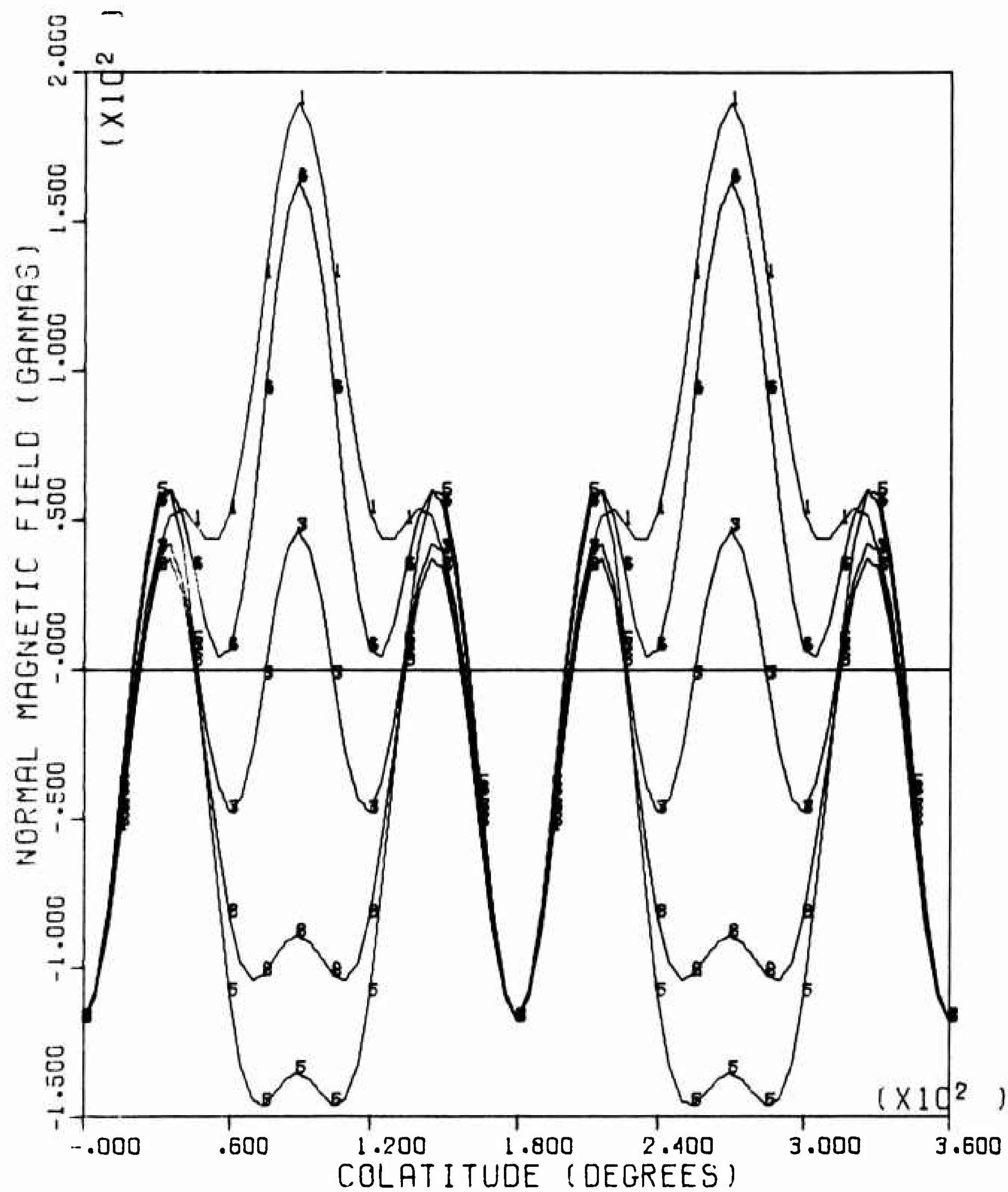


FIG. 31 MULTIPOLE COMPONENT OF DEGREE 6

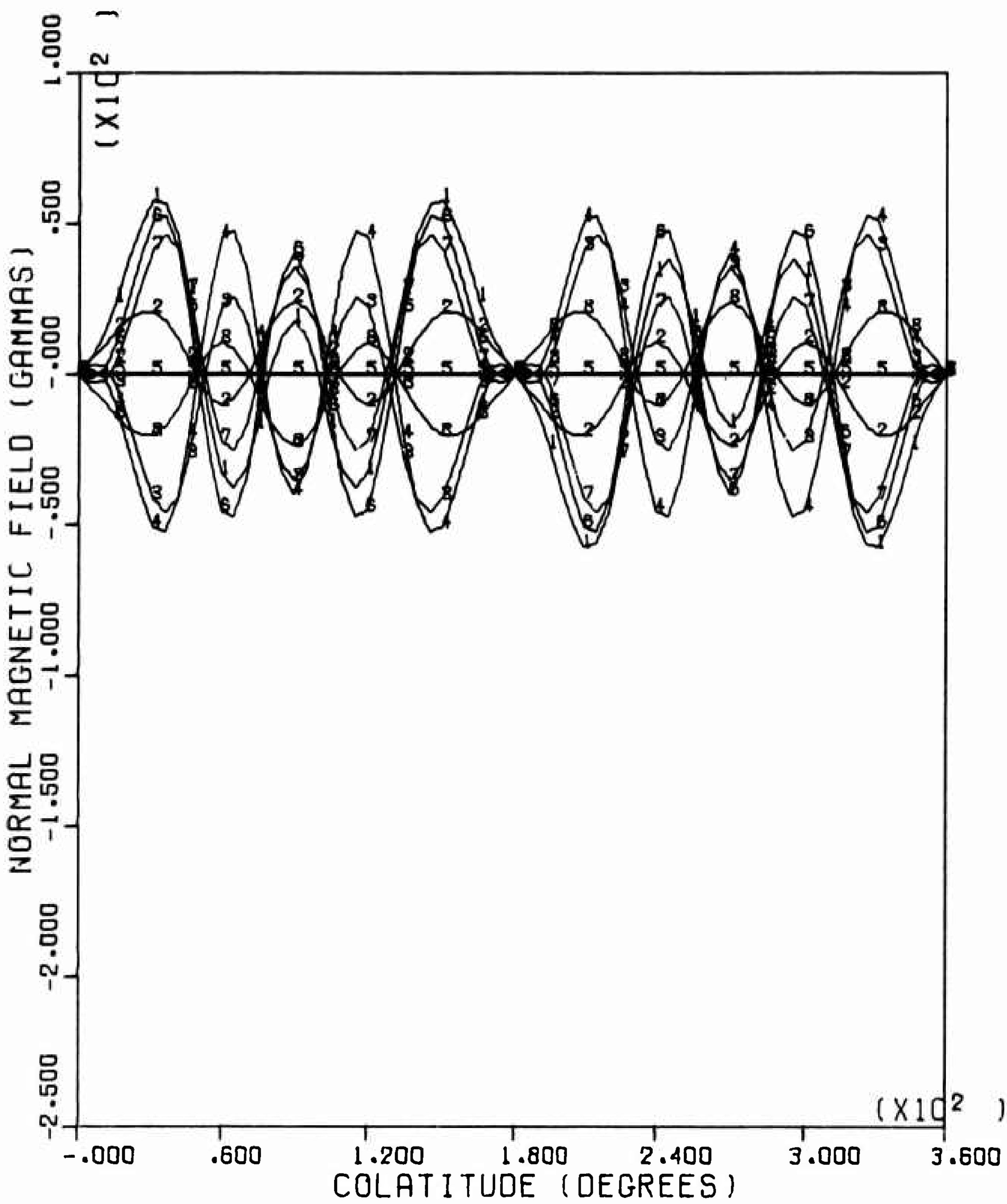


FIG. 32 MULTIPOLE COMPONENT OF DEGREE 7

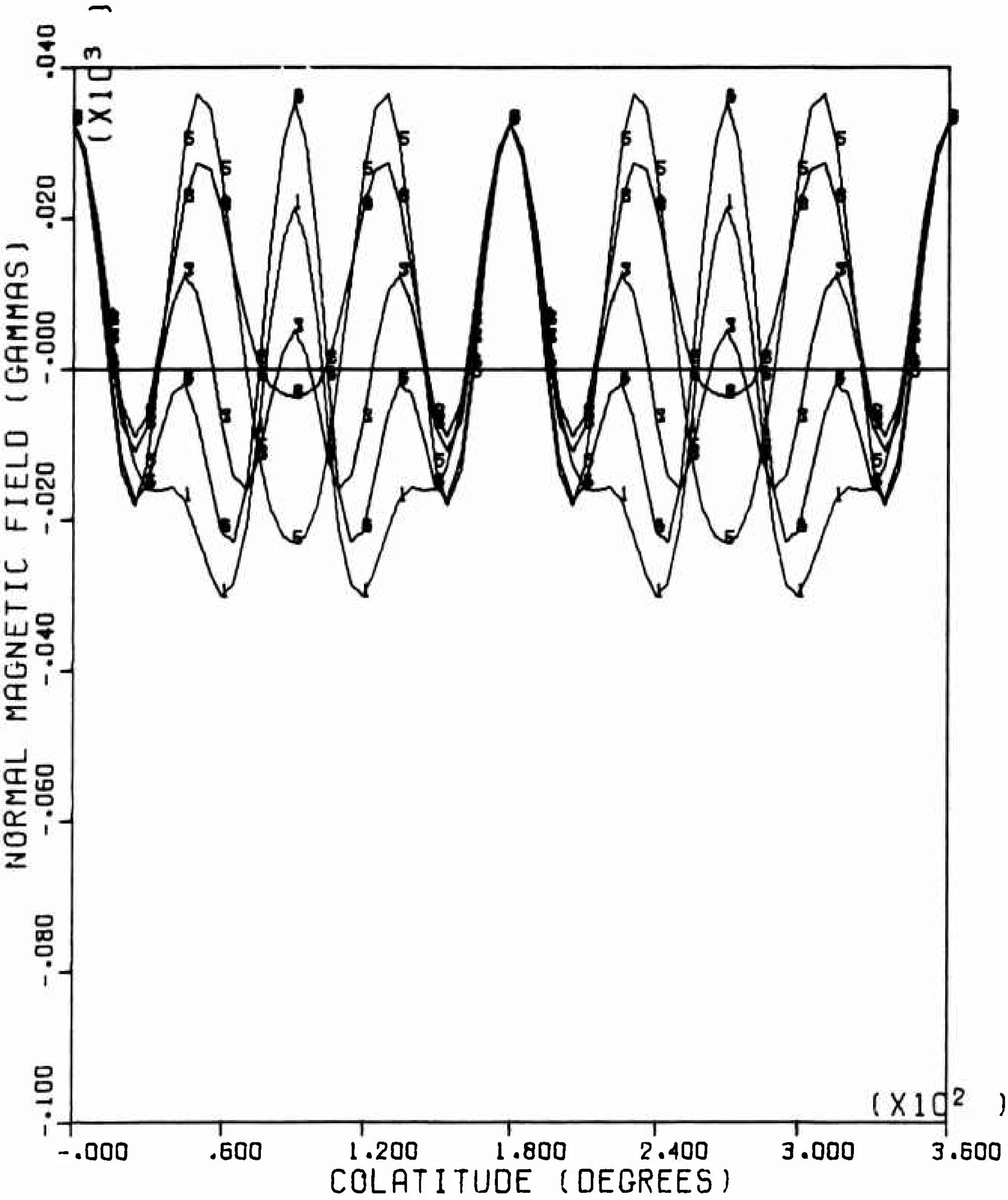


FIG. 33 MULTIPOLE COMPONENT OF DEGREE 8

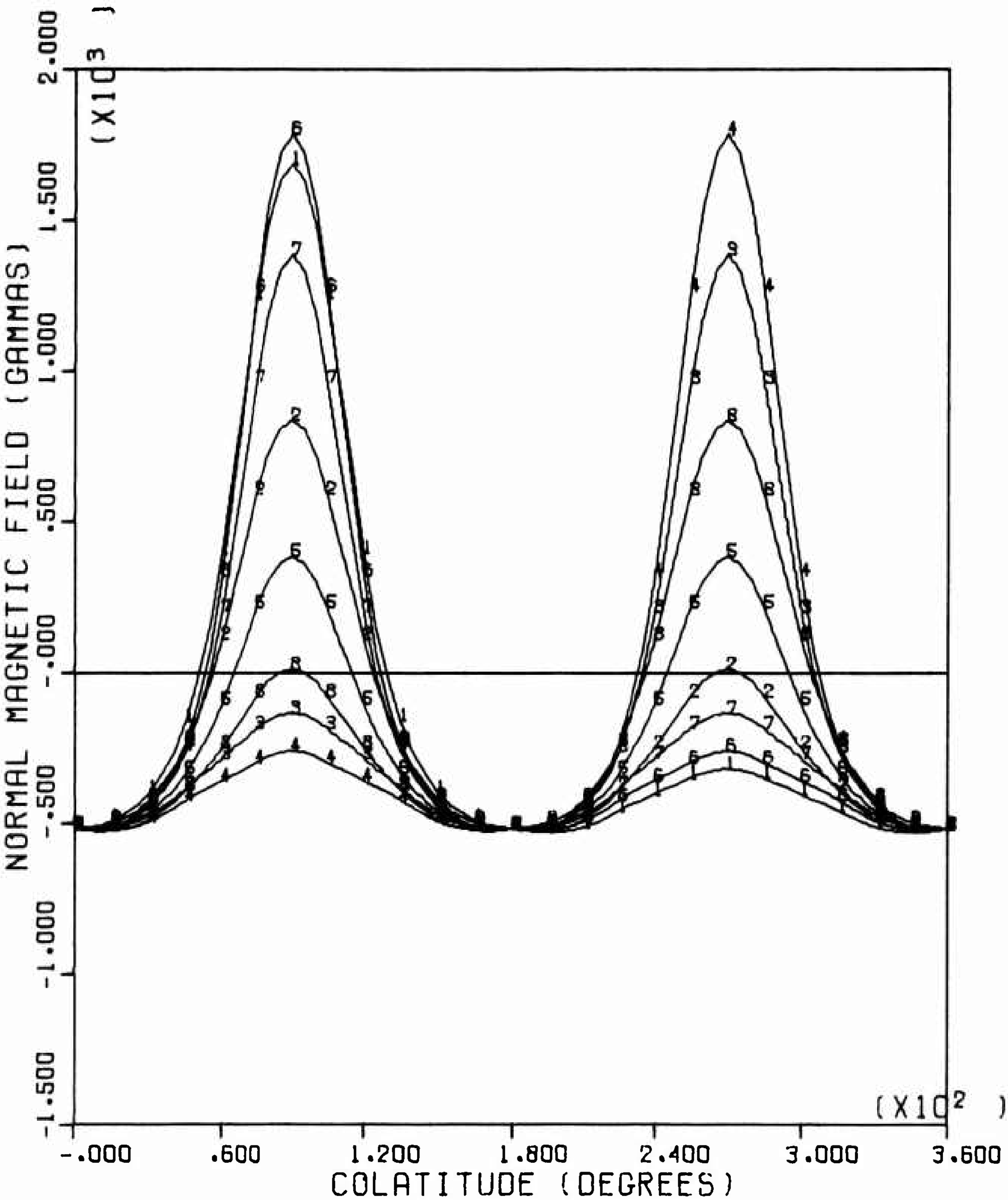


FIG. 34 SUMMATION OF FIRST EIGHT MULTIPOLE COMPONENTS

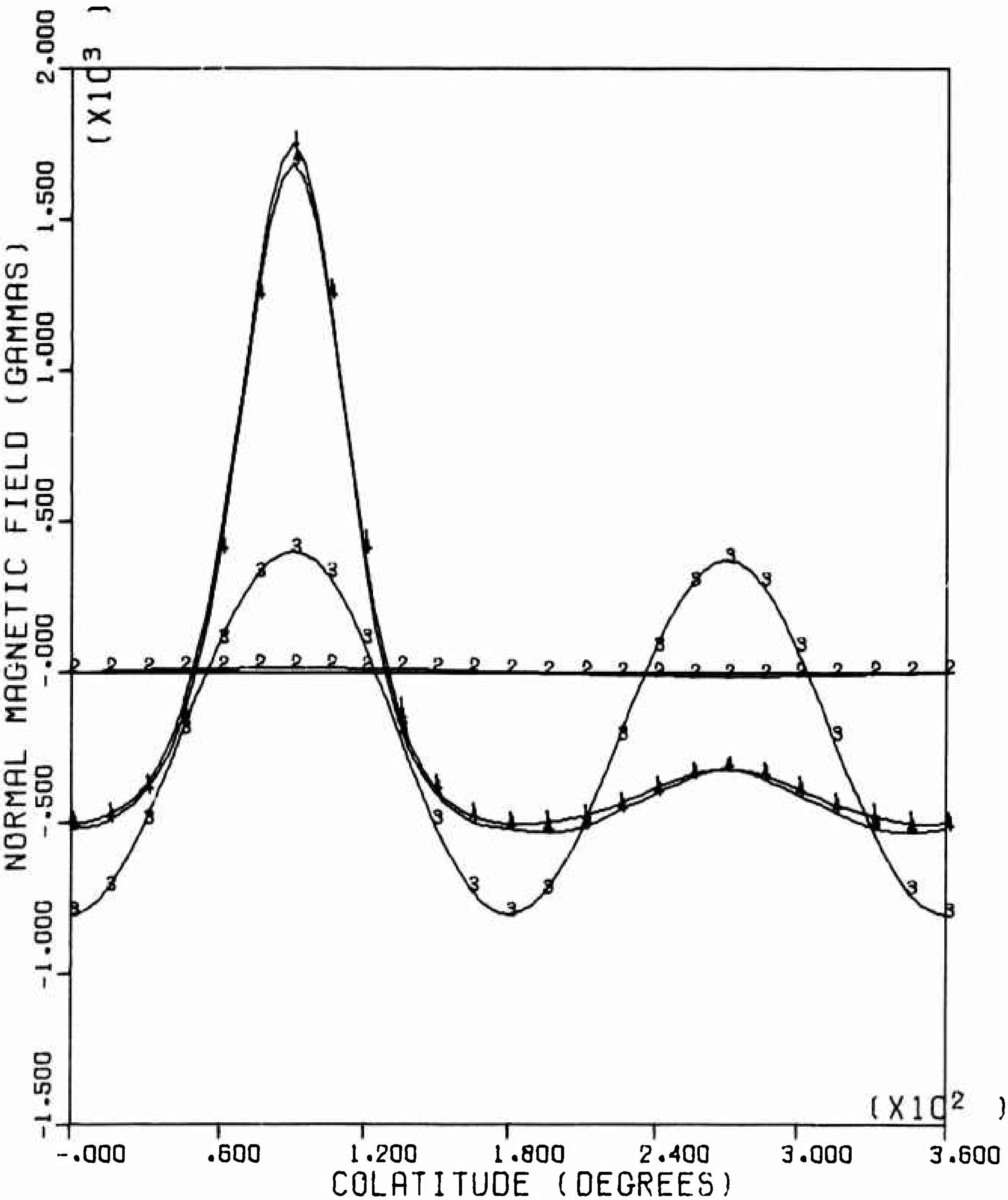


FIG. 35 INDIVIDUAL CURVES DEMONSTRATING APPROXIMATION ACCURACY

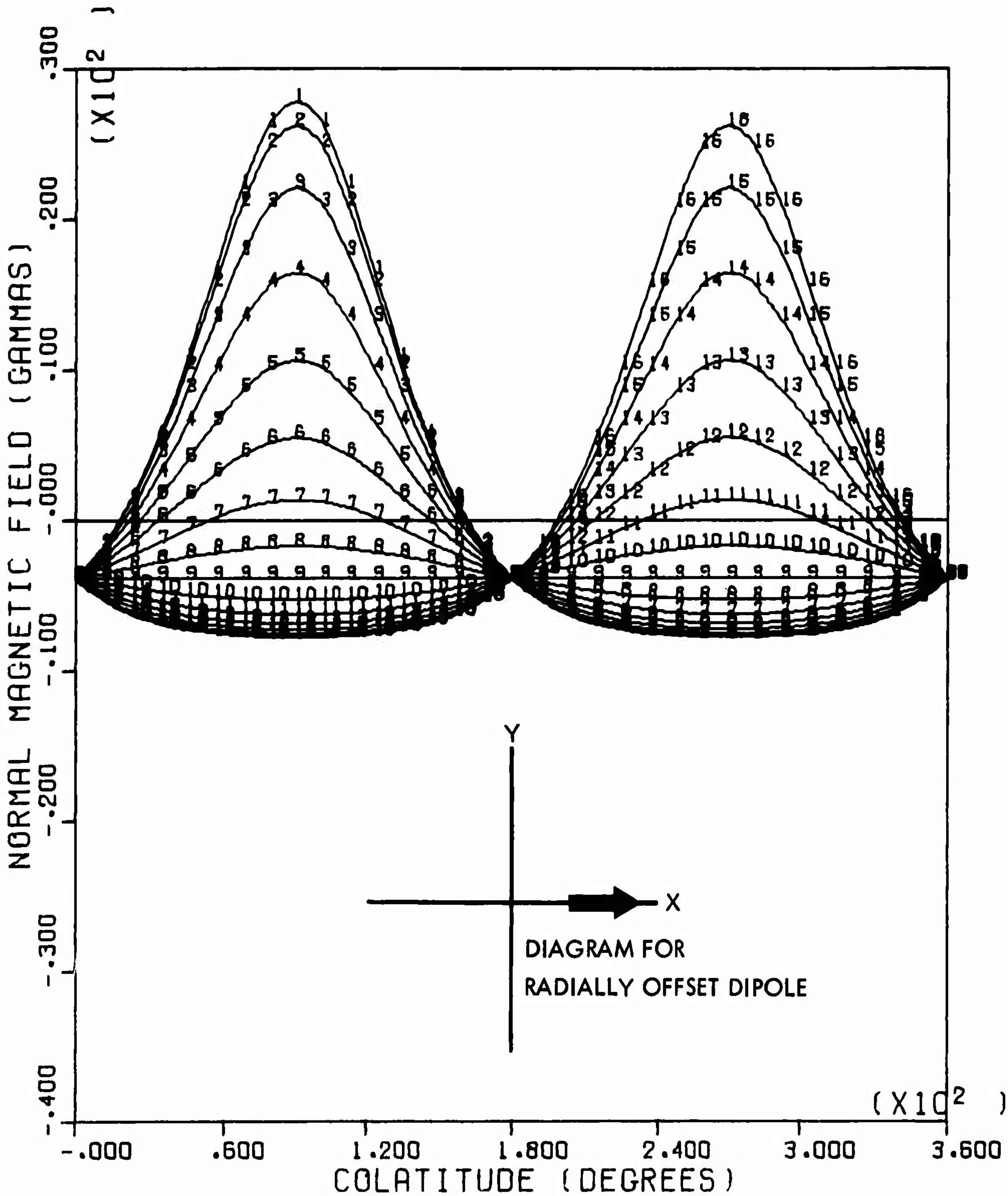


FIG. 36 DATA CURVES FOR RADIALLY OFFSET DIPOLE

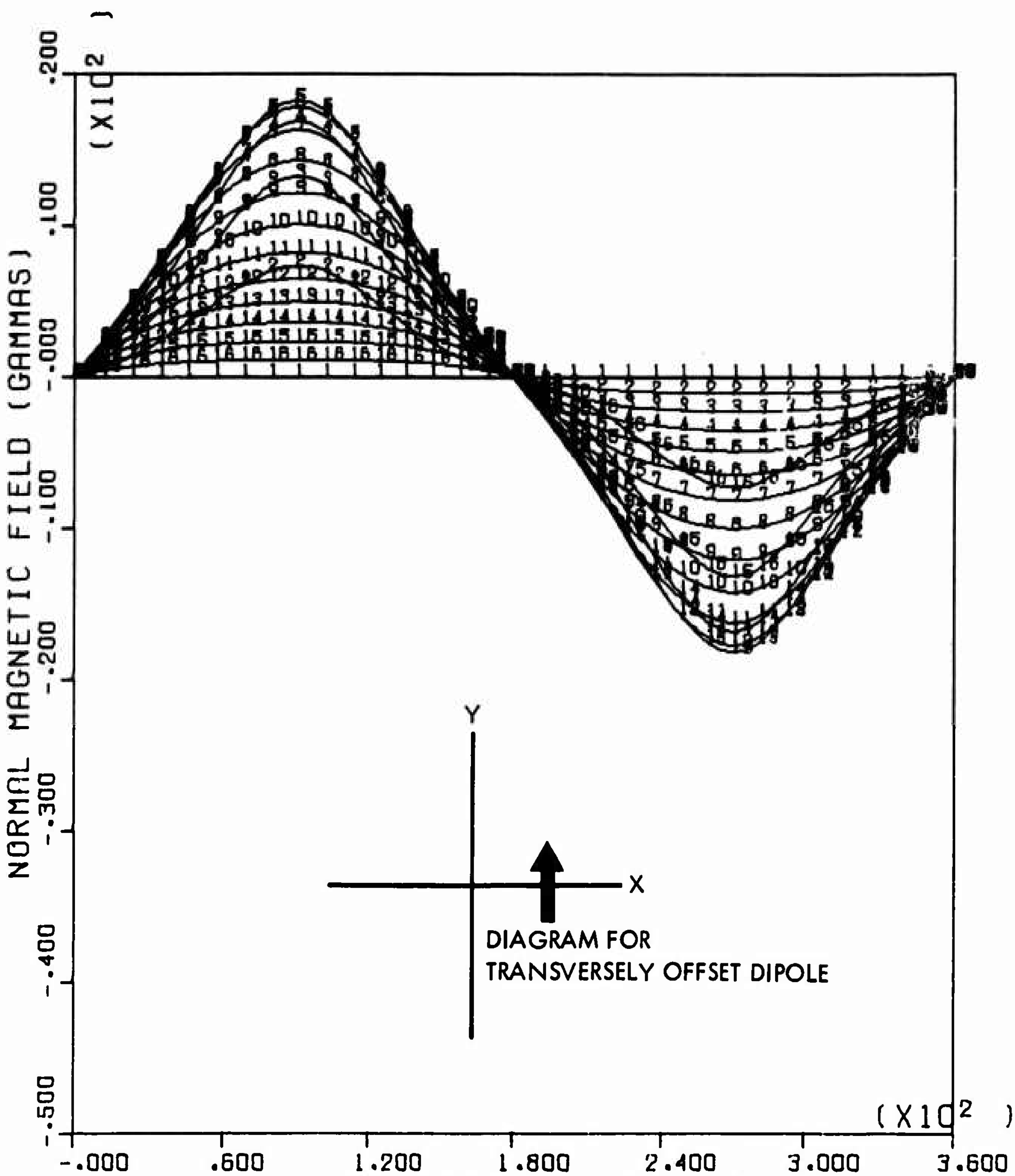


FIG. 37 DATA CURVES FOR TRANSVERSELY OFFSET DIPOLE

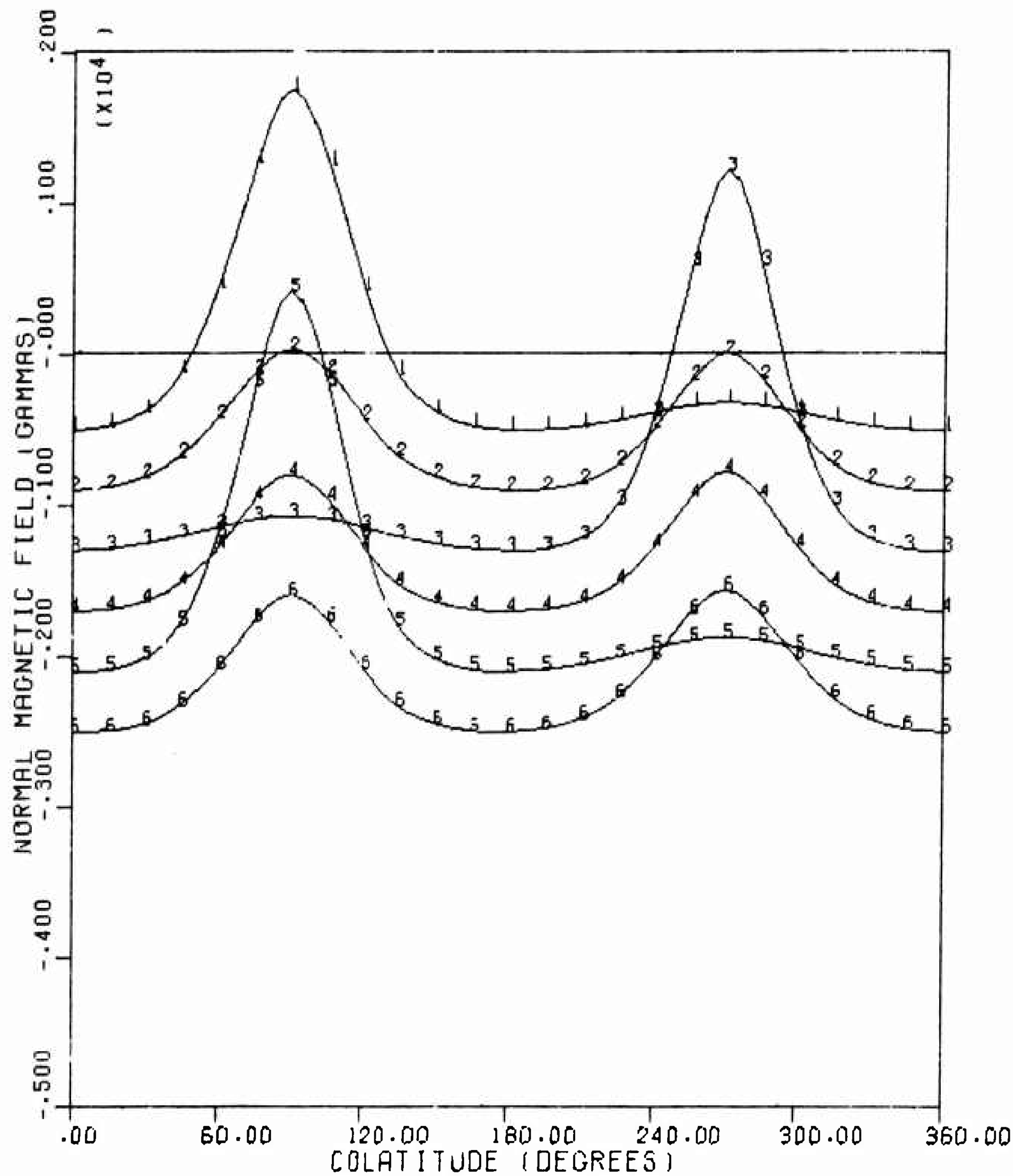


FIG. 38 SAMPLE GRAPH FROM ELECTROSTATIC PLOTTER

APPENDIX A

GLOSSARY OF SYMBOLS AND TERMS

A(I) - is the spherical harmonic coefficient A^{I}_{NN} in the form used by the computer programs. Eqs. (5) and (8) define the relationships.

A(n,m) - is the spherical harmonic coefficient A^m_n as an element of a two-dimensional array.

A^m_n - is the spherical harmonic coefficient as defined by Eq. (5).

B(I) - is the spherical harmonic coefficient B^{I}_{NN} in the form used by the computer programs. Eqs. (5) and (8) define the relationships.

B(n,m) - is the spherical harmonic coefficient B^m_n as an element of a two-dimensional array.

B^m_n - is the spherical harmonic coefficient as defined by Eq. (5).

C - is the constant factor in the set of weighting factors $\{Y(I)\}$. C has the value (π/NO) representing the interval of longitude. (see Eqs. (14) and (15)).

C(I) - is the Fourier coefficient representing the amplitude of the I<u>th</u> term in the expansion of the data curves. Eqs. (32) and (34) define the relationships.

C^m_n - is the coefficient used in Eq. (26b). It originates in the identity

$$\frac{dP^m_n(\cos\theta)}{d\theta} = m\cos\theta\, P^m_n(\cos\theta)/\sin\theta - C^m_n\, P^{m+1}_n(\cos\theta)$$

where

$$C^m_n = \left[(n-m)(n+m+1)\right]^{\frac{1}{2}} \text{ for } (m > 0)$$
$$= \left[n(n+1)/2\right]^{\frac{1}{2}} \text{ for } (m = 0)$$

CS - is the parameter in the program SA3024 that represents the data value resulting from a static measurement by the magnetic sensor with a calibration signal turned on.

CV - is the value (in gammas) of the calibration signal. The values of CS, CV, and CZ are used to convert the measured data into gammas, i.e., each data value is multiplied by CV/(CS-CZ).

CZ - is the parameter in the program SA3024 that represents the data value resulting from a static measurement by the magnetic sensor with the calibration signal turned off.

C0 - ($= C_o$) is the Fourier coefficient representing the constant component of the data curves. It is defined in Eq. (30).

$C1_j$ - is the Fourier coefficient for the jth cosine term in the expansion of the data curves. It is defined in Eq. (31b.).

C6 - is the parameter in the BASIC program SA1024 which corresponds to CV.

C7 - is the parameter in the BASIC program SA1024 which corresponds to CS.

C8 - is the parameter in the BASIC program SA1024 which corresponds to CZ.

D - is the magnitude of the dipole moment $\bar{D}$. It is equal to $(D_1^2 + D_2^2 + D_3^2)^{\frac{1}{2}}$.

$\bar{D}$ - is the dipole moment vector (D_1, D_2, D_3) defined by Eq. (1).

D(I) - is the Ith factor of the set of weighting factors {Y(I)}. It is a measure of the interval of colatitude assigned to the data points {F(I,J), J = 1,2,···,NO}. (See Eqs. (14) through (18).)

D_1, D_2, D_3 - are the x, y, and z-components, respectively, of the dipole moment $\bar{D}$.

DEV - is the maximum difference between data points of a measured curve and the corresponding points of the curve defined by the approximated Fourier coefficients.

EA - is the parameter in programs SA4024 and SA5024 that represents the angular error (in degrees) to be randomly inserted into the simulated measurement data. It represents the measurement position errors.

ED - is the parameter in programs SA4024 and SA5024 that represents a constant offset (in gammas) of the measurement instrumentation. The analysis procedure handles this value as the magnetic field from a monopole moment.

EG - is the parameter in programs SA4024 and SA5024 that represents the error (in gammas) to be randomly inserted into the simulated measurement data. It represents instrumentation inaccuracies.

F(I) - is the argument representing the x, y, and z-component of the magnetic field vector computed by the subroutine AMPFLD. F(I), for I = 1,2,$\cdots$,N is also the array of data for the subroutine FNCTON.

F(IJ) - is the single-dimensioned array of data values representing the normal component of the magnetic field on the surface of the measurement sphere surrounding the satellite. The relationship is F(IJ) = f(θ(I), φ(J)) for IJ = I + (J - 1) $\cdot$ N1, I = 1,2,$\cdots$,N1, and J = 1,2,$\cdots$,NO.

F(I,J) - is the two-dimensional array of numbers representing discrete values of the normal component of the magnetic field on the surface of the measurement sphere surrounding the satellite. The values are actually stored in the single-dimensioned array F(IJ) = F(I,J) = f(θ(I), φ(J)) for IJ = I + (J - 1) $\cdot$ N1, I = 1,2,$\cdots$,N1, and J = 1,2,$\cdots$,NO.

$f(\theta,\varphi)$ - represents the normal component of the total magnetic field on the surface of the measurement sphere of radius R1 surrounding the satellite.

$f_n(\theta,\varphi)$ - represents the normal component of the magnetic field on the surface of the measurement sphere from the multipole moment of degree n.

$f_{nj}(\theta,\varphi)$ - represents the normal component of the magnetic field on the surface of the measurement sphere from the jth multipole magnet of degree n.

F9 - is the format for reading and printing the spherical coefficients A(I) and B(I), e.g., F9 = (1H ,7E10.4).

$\bar{H}(\bar{R})$ - represents the total magnetic field vector at the point $\bar{R}$.

$\bar{H}_n(\bar{R})$ - represents the magnetic field vector at the point $\bar{R}$ from a multipole magnet of degree n.

$\bar{i}$ - represents the unit vector along the x-axis.

IP - is the parameter in the program SA4024 that determines whether or not the simulated measurement data is to be interpolated and plotted. IP $\neq$ 0 means that the data will be interpolated and plotted.

IP1 - is the parameter in the programs SA3024 and SA5024 which corresponds to IP.

IP2 - is the parameter in the programs SA3024 and SA5024 that determines whether or not the measurement data is to be printed. IP2 $\neq$ 0 means that the data will be printed.

IR - is the parameter in the programs SA2024 and SA3024 that determines whether or not to read the measurement data from a data file. IR $\neq$ 0 means that the data will be read from the file DAT024.

IW - is the parameter in programs SA3024 and SA5024 that determines which integrating scheme is to be used. IW = 0 means that the exact, algebraic scheme is to be used.

$\bar{j}$ - represents the unit vector along the y-axis.

$\bar{k}$ - represents the unit vector along the z-axis.

m - is the order of the coefficients A_n^m and B_n^m, and the polynomials $P_n^m(\cos\theta)$ and $P_{n,m}(\cos\theta)$.

M - is the program parameter representing the order of the coefficients and polynomials associated with multipole magnets.

n - is the degree of the coefficients A_n^m and B_n^m, and the polynomials $P_n^m(\cos\theta)$ and $P_{n,m}(\cos\theta)$.

N - is the program parameter representing the degree of the coefficients and polynomials associated with multipole magnets. N is also used to define the number of distinct data points for the subroutine FNCTON.

NO - is the number of curves of data. It is an even integer (≤ 16) corresponding to the number of great circles of data on the measurement sphere.

N1 - is the number of equally spaced data points per curve ranging from 0 through 360 degrees in colatitude. N1 is an odd integer (≤ 33).

N2 - is the parameter in the BASIC program SA1024 which corresponds to IP2. N2 is also used in the subroutine FNCTON to designate the number of Fourier terms that produces the minimum error (DEV) between the data and the computed curve.

N3 - is the number of distinct factors $\{D(I)\}$ of the weights $\{Y(I)\}$. N3 is the largest integer that is less than or equal to $(N1 + 3)/4$, i.e., $N3 = \left[\max_{1 \leq i} i \text{ such that } i \leq (N1 + 3)/4\right]$.

$\bar{n}(\theta,\varphi)$ - is the unit normal vector on (exterior to) the surface of the measurement sphere at the point $(R1, \theta, \varphi)$. It has rectilinear components $(\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$.

NH - is the parameter in the program SA3024 that represents the highest degree spherical harmonic term to be computed from the data (NH = 1 for dipoles, 2 for quadrupoles, etc.). NH is also the parameter in the program SA4024 that represents the total number of different harmonics (degrees) of multipole magnets to be considered.

NH1 - is the parameter in the program SA5024 that represents the total number of different harmonics (degrees) of multipole magnets to be considered.

NH2 - is the parameter in the program SA5024 that represents the highest degree spherical harmonic term to be computed from the data (NH2 = 1 for dipoles, 2 for quadrupoles, etc.)

NM - is the parameter in the programs SA4024 and SA5024 that represents the number of multipole magnets with harmonic number (degree) NN.

NN - is the parameter in the programs SA4024 and SA5024 that represents the harmonic number (degree) for the multipole data being read in (NN = 1 for dipole, 2 for quadrupoles, etc.).

P - represents the multipole position vector in the programs SA4024 and SA5024. It also represents the weighting factors in the subroutine AMPMNT.

P4 - is the parameter in programs SA3024, SA4024, and SA5024 that represents the scale factor in gammas/inch for the y-axis if the data is to be plotted. If PY = 0.0, and if the data is to be plotted, a suitable factor will be computed from the data.

$P_{n,m}(\cos\theta)$ - represents the associated Legendre polynomial of degree n and order m. Reference (b) gives the definition as $P_{n,m}(\cos\theta) \equiv \sin^m\theta\ d^mP_n(\cos\theta)/d(\cos\theta)^m$ where. $P_n(\cos\theta)$ is the regular Legendre polynomial of degree n. (See also Eq. (2) of Appendix K.)

$P_n^m(\cos\theta)$ - represents the Schmidt polynomial of degree n and order m. They are defined in Eq. (3) of Appendix K.

PHI(I) - is the Fourier coefficients in the subroutine FNCTON which represent the phase angles defined by Eqs. (31) and (33).

$Q_{11},\cdots,Q_{33}$ - represent the coefficients of the quadrupole moment term in rectilinear coordinates.

$\bar{r}$ - represents an element of the orthonormal set of vectors defined by Eq. (23).

$\bar{R}$ - represents the position vector for a point at which the magnetic field is to be computed by the subroutine AMPFLD. Coordinate transformations are included in Figure 1.

$\bar{R}_1$ - represents the vector between a multipole position and the point at which the magnetic field is to be computed by the subroutine AMPFLD.

R1 - is the parameter in the programs SA1024,$\cdots$,SA5024 that represents the radius of the measurement sphere in inches.

$S1_j$ - is the Fourier coefficient for the jth sine term in the expansion of the data curves. It is defined by Eq. (31a).

X(I) - represents the surface area or weight assigned to the data points {F(I,J),J = 1,2,···,NO} when the geometric or approximate integrating scheme is used. It is defined in Eq. (4).

Y(I) - represents the surface area or weight assigned to the data points {F(I,J),J = 1,2,···,NO} when either integration scheme is used.

YDIST - is the parameter in the subroutine DATPLT that corresponds to the parameter PY.

θ - represents the spherical coordinate of colatitude defined in terms of rectilinear coordinates as $\theta = \tan^{-1}\left[(x^2 + y^2)^{\frac{1}{2}}/z\right]$.

$\bar{\theta}$ - represents an element of the orthonormal set of vectors defined by Eq. (23).

θ(I) - represents the Ith element in the set of values of colatitude at which the data F(I,J) is recorded.

φ - represents the spherical coordinate of longitude defined in terms of rectilinear coordinates as $\varphi = \tan^{-1}(x/y)$.

$\bar{\varphi}$ - represents an element of the orthonormal set of vectors defined by Eq. (23).

φ(J) - represents the jth element in the set of values of longitude at which the data F(I,J) is recorded.

APPENDIX B

LISTING OF SA1024

```
00010 REM PROGRAM SATDPL                                          SA100010
00020 DIM R(3),W(3),F(300)                                        SA100020
00030 LET P1=3.14159265                                           SA100030
00040 READ NO,N1                                                  SA100040
00050 IF NO<>0 THEN 00070                                         SA100050
00060 STOP                                                        SA100060
00070 READ R1,C6                                                  SA100070
00080 LET R1=R1*2.54                                              SA100080
00090 PRINT "ENTER 1 OR 0 TO INDICATE DATA PRINT OR NOT"          SA100090
00100 READ N2                                                     SA100100
00110 LET D0=2.*P1/(N1-1)                                         SA100110
00120 FOR I=1 TO 3 STEP 1                                         SA100120
00130 LET W(I)=0                                                  SA100130
00140 LET R(I)=0                                                  SA100140
00150 NEXT I                                                      SA100150
00160 LET N3=NO*N1                                                SA100160
00170 READ C7,C8                                                  SA100170
00180 FOR I=1 TO N3 STEP 1                                        SA100180
00190 READ F(I)                                                   SA100190
00200 NEXT I                                                      SA100200
00210 PRINT                                                       SA100210
00220 GOSUB 01160                                                 SA100220
00230 IF N2=0 THEN 00250                                          SA100230
00240 GOSUB 01000                                                 SA100240
00250 PRINT                                                       SA100250
00260 FOR K0=1 TO NO STEP 1                                       SA100260
00270 LET O3=P1*(K0-1)/NO                                         SA100270
00280 LET S3=SIN(O3)                                              SA100280
00290 LET C3=COS(O3)                                              SA100290
00300 FOR L0=1 TO N1 STEP 1                                       SA100300
00310 LET K1=L0+(K0-1)*N1                                         SA100310
00320 LET F1=F(K1)*1.E-5                                          SA100320
00330 LET O4=2.*P1*(L0-1)/(N1-1)                                  SA100330
00340 LET S4=SIN(O4)                                              SA100340
00350 LET C4=COS(O4)                                              SA100350
00360 LET P5=(SIN(D0/4)**2)/(2.*NO)                               SA100360
00370 IF L0=1 THEN 00430                                          SA100370
00380 IF L0=N1 THEN 00430                                         SA100380
00390 IF L0<>(N1+1)/2 THEN 00420                                  SA100390
00400 LET P5=P5*2.                                                SA100400
00410 GO TO 00430                                                 SA100410
00420 LET P5=ABS(SIN(O4)*SIN(D0/2))/(2.*NO)                       SA100420
00430 LET R(1)=C3*S4                                              SA100430
00440 LET R(2)=S3*S4                                              SA100440
00450 LET R(3)=C4                                                 SA100450
00460 FOR I=1 TO 3 STEP 1                                         SA100460
00470 W(I)=W(I)+(F1*P5)*R(I)                                      SA100470
00480 NEXT I                                                      SA100480
```

```
00490 NEXT LO                                                          SA100490
00500 NEXT KO                                                          SA100500
00510 FOR I=1 TO 3 STEP 1                                              SA100510
00520 LET W(I)=(1.5*(R1**3))*W(I)                                      SA100520
00530 NEXT I                                                           SA100530
00540 LET R(1)=SQR(W(1)**2+W(2)**2+W(3)**2)                            SA100540
00550 LET S8=W(2)                                                      SA100550
00560 LET S9=W(1)                                                      SA100560
00570 GOSUB 01670                                                      SA100570
00580 LET R(2)=C5*180./P1                                              SA100580
00590 LET S8=SQR(W(1)**2+W(2)**2)                                      SA100590
00600 LET S9=W(3)                                                      SA100600
00610 GOSUB 01670                                                      SA100610
00620 LET R(3)=C5*180./P1                                              SA100620
00630 PRINT "(DX,DY,DZ) = ";W(1);W(2);W(3)                             SA100630
00640 PRINT "(D ,02,01) = ";R(1);R(2);R(3)                             SA100640
00650 GOSUB 01320                                                      SA100650
00660 GO TO 00040                                                      SA100660
00670                                                                  SA100670
00680 REM DATA NO,N1                                                   SA100680
00690 DATA 6,25                                                        SA100690
00700 REM DATA R1,C6                                                   SA100700
00710 DATA 96.0,1.0                                                    SA100710
00720 REM DATA N2                                                      SA100720
00730 DATA 1                                                           SA100730
00740 REM DATA C7,C8,(F(I),I=1,N3)                                     SA100740
00750 DATA    1.0,     0.0,                                            SA100750
00760 DATA -504.3,  -475.5,  -381.5,  -123.3,   438.4,  1273.7,  1749.2,SA100760
00770 DATA 1259.6,   431.3,  -125.7,  -382.1,  -476.3,  -502.7,  -499.8,SA100770
00780 DATA -476.5,  -436.5,  -386.3,  -341.0,  -322.6,  -339.4,  -386.5,SA100780
00790 DATA -435.9,  -476.7,  -499.0,  -503.7,  -902.8,  -881.2,  -814.6,SA100790
00800 DATA -675.4,  -420.8,  -118.9,    27.2,  -119.7,  -426.7,  -672.7,SA100800
00810 DATA -815.5,  -880.6,  -904.1,  -897.6,  -851.0,  -728.7,  -483.8,SA100810
00820 DATA -164.5,     7.2,  -160.4,  -480.6,  -728.9,  -850.4,  -897.1,SA100820
00830 DATA -903.7, -1302.5, -1287.7, -1254.4, -1206.6, -1148.0, -1096.8,SA100830
00840 DATA-1077.9, -1097.1, -1148.4, -1206.6, -1255.4, -1288.3, -1303.0,SA100840
00850 DATA-1292.6, -1219.0,  -990.5,  -394.6,   589.2,  1219.4,   595.2,SA100850
00860 DATA -406.4,  -993.0, -1219.6, -1291.0, -1304.2, -1703.5, -1691.6,SA100860
00870 DATA-1642.3, -1519.4, -1281.9,  -961.8,  -800.9,  -966.5, -1277.0,SA100870
00880 DATA-1520.1, -1642.5, -1692.0, -1704.1, -1690.6, -1640.8, -1515.1,SA100880
00890 DATA-1274.8,  -946.5,  -779.8,  -946.2, -1274.5, -1518.1, -1641.6,SA100890
00900 DATA-1691.1, -1703.7, -2102.4, -2092.4, -2019.9, -1788.4, -1202.2,SA100900
00910 DATA -211.0,   418.3,  -207.2, -1209.2, -1792.5, -2019.2, -2091.1,SA100910
00920 DATA-2103.6, -2088.4, -2056.8, -2005.6, -1947.4, -1898.5, -1877.5,SA100920
00930 DATA-1897.9, -1947.7, -2006.3, -2054.5, -2088.6, -2103.4, -2502.4,SA100930
00940 DATA-2497.0, -2452.0, -2327.8, -2086.3, -1770.2, -1598.7, -1762.8,SA100940
00950 DATA-2082.9, -2329.9, -2448.4, -2497.0, -2503.6, -2481.7, -2415.6,SA100950
00960 DATA-2274.3, -2022.5, -1713.6, -1566.7, -1720.3, -2020.7, -2271.2,SA100960
00970 DATA-2414.6, -2481.5, -2503.5                                    SA100970
00980 DATA 0,  0,                                                      SA100980
00990                                                                  SA100990
01000 REM SUBROUTINE PRINTF                                            SA101000
01010 FOR KO= 1 TO NO STEP 1                                           SA101010
01020 LET K2=1+(KO-1)*(N1)                                             SA101020
01030 LET K3=N1+(KO-1)*(N1)                                            SA101030
01040 PRINT                                                            SA101040
01050 PRINT "CURVE NO."; KO                                            SA101050
01060 FOR K4= K2 TO K3 STEP 1                                          SA101060
01070 IF F(K4)<1E-2 THEN 01090                                         SA101070
01080 GO TO 01110                                                      SA101080
```

```
01090 IF F(K4)<-1E-2 THEN 01110                                        SA101090
01100 LET F(K4)=0.0                                                    SA101100
01110 PRINT F(K4),                                                     SA101110
01120 NEXT K4                                                          SA101120
01130 NEXT KO                                                          SA101130
01140 RETURN                                                           SA101140
01150                                                                  SA101150
01160 REM SUBROUTINE SHIFT                                             SA101160
01170 LET Z1=(F(1)+F(N1))/2.                                           SA101170
01180 LET N3=(N1+1)/2                                                  SA101180
01190 LET Z2=F(N3)                                                     SA101190
01200 FOR KO= 2 TO NO STEP 1                                           SA101200
01210 LET K2=1+(KO-1)*N1                                               SA101210
01220 LET K3=KO*N1                                                     SA101220
01230 LET K4=N3+(KO-1)*N1                                              SA101230
01240 LET C1=(2.*Z1+Z2-F(K2)-F(K4)-F(K3))/3.                           SA101240
01250 FOR LO=1 TO N1 STEP 1                                            SA101250
01260 LET K4=LO+(KO-1)*N1                                              SA101260
01270 LET F(K4)=(F(K4)+C1)*C6/(C7-C8)                                  SA101270
01280 NEXT LO                                                          SA101280
01290 NEXT KO                                                          SA101290
01300 RETURN                                                           SA101300
01310                                                                  SA101310
01320 REM SUBROUTINE PRINT1                                            SA101320
01330 DIM S(3)                                                         SA101330
01340 LET P1=3.14159265                                                SA101340
01350 FOR I=1 TO 3 STEP 1                                              SA101350
01360 LET S(I)=-W(I)                                                   SA101360
01370 NEXT I                                                           SA101370
01380 LET R(1)=SQR(S(1)**2+S(2)**2+S(3)**2)                            SA101380
01390 LET S8=S(2)                                                      SA101390
01400 LET S9=S(1)                                                      SA101400
01410 GOSUB 01670                                                      SA101410
01420 LET R(2)=C5*180./P1                                              SA101420
01430 LET S8=SQR(S(1)**2+S(2)**2)                                      SA101430
01440 LET S9=S(3)                                                      SA101440
01450 GOSUB 01670                                                      SA101450
01460 LET R(3)=C5*180./P1                                              SA101460
01470 LET W3=ABS(W(3))                                                 SA101470
01480 LET W4=.2*W3                                                     SA101480
01490 LET A1=SQR(S(1)**2+S(2)**2)                                      SA101490
01500 LET A2=.2*A1                                                     SA101500
01510 PRINT "     THE COMPENSATING MAGNET FOR THE XY-PLANE SHOULD";    SA101510
01520 PRINT " BE"A1"GAUSS-"                                            SA101520
01530 PRINT "CENTIMETER-CUBED WITH THE NORTH POLE POINTING"R(2);       SA101530
01540 PRINT "DEGREES FROM +X."                                         SA101540
01550 PRINT "(THE MAGNET SHOULD READ"A2"GAMMA AT ONE METER.)"          SA101550
01560 PRINT "     THE COMPENSATING MAGNET FOR THE Z-AXIS SHOULD BE";   SA101560
01570 PRINT W3" GAUSS-"                                                SA101570
01580 IF S(3)=>0. THEN 01620                                           SA101580
01590 PRINT "CENIMETER-CUBED WITH THE NORTH POLE POINTING ";           SA101590
01600 PRINT "TOWARDS -Z (THE MAGNET "                                  SA101600
01610 GO TO 01640                                                      SA101610
01620 PRINT "CENTIMETER-CUBED WITH THE NORTH POLE POINTING ";          SA101620
01630 PRINT "TOWARDS +Z (THE MAGNET) "                                 SA101630
01640 PRINT "SHOULD READ"W4"GAMMA AT ONE METER."                       SA101640
01650 RETURN                                                           SA101650
01660                                                                  SA101660
01670 REM SUBROUTINE ARCTAN                                            SA101670
01680 IF S9=0 THEN 01750                                               SA101680
```

```
01690 LET C5=S8/S9                                    SA101690
01700 IF S9>0 THEN 01730                              SA101700
01710 LET C5=ATN(C5)+P1                               SA101710
01720 GO TO 01790                                     SA101720
01730 LET C5=ATN(C5)                                  SA101730
01740 GO TO 01790                                     SA101740
01750 IF S8<0 THEN 01780                              SA101750
01760 LET C5=P1/2                                     SA101760
01770 GO TO 01790                                     SA101770
01780 LET C5=3*P1/2                                   SA101780
01790 RETURN                                          SA101790
01800 END                                             SA101800
```

APPENDIX C

LISTING OF SA2024

```
      PROGRAM SATDPL(INPUT,OUTPUT,DAT024,TAPE7=DAT024)
      DIMENSION R(3),W(3),F(800)
      P1=3.14159265358979
      REWIND 7
   10 READ 01, NO,N1,IR
      IF(NO.EQ.0) STOP
      READ 03, R1,C6
      R1=R1*2.54
      PRINT 02
      READ 01, N2
      D0=2.*P1/FLOAT(N1-1)
      DO 20 I=1,3
      W(I)=0
      R(I)=0
   20 CONTINUE
      N3=NO*N1
      IF(IR.EQ.0) READ 03, C7,C8,(F(I),I=1,N3)
      IF(IR.EQ.0) WRITE(7,03) C7,C8,(F(I),I=1,N3)
      IF(IR.NE.0) READ(7,03) C7,C8,(F(I),I=1,N3)
      CALL SHIFT(F,NO,N1,C6,C7,C8)
      IF(N2.NE.0) CALL PRINTF(F,NO,N1)
      DO 60 KO=1,NO
      O3=P1*(FLOAT(KO-1)/FLOAT(NO))
      S3=SIN(O3)
      C3=COS(O3)
      DO 60 LO=1,N1
      KL=LO+(KO-1)*N1
      F1=F(KL)*1.E-5
      O4=2.*P1*(FLOAT(LO-1)/FLOAT(N1-1))
      S4=SIN(O4)
      C4=COS(O4)
      P5=(SIN(D0/4)**2)/(2.*FLOAT(NO))
      IF(LO.EQ.1.OR.LO.EQ.N1) GO TO 40
      IF(LO.NE.(N1+1)/2) GO TO 30
      P5=P5*2.
      GO TO 40
   30 P5=ABS(SIN(O4)*SIN(D0/2))/(2.*FLOAT(NO))
   40 R(1)=C3*S4
      R(2)=S3*S4
      R(3)=C4
      DO 50 I=1,3
   50 W(I)=W(I)+(F1*P5)*R(I)
   60 CONTINUE
      DO 70 I=1,3
      W(I)=(1.5*(R1**3))*W(I)
   70 CONTINUE
      CALL SPCOOR(W,R)
      PRINT 04, W(1),W(2),W(3)
```

```
      PRINT 05, R(1),R(2),R(3)
      CALL PRINTI(W)
      GO TO 10
   01 FORMAT(14I5)
   02 FORMAT(44HOENTER 1 OR 0 TO INDICATE DATA PRINT OR NOT9)
   03 FORMAT(9F6.1)
   04 FORMAT(14H (DX,DY,DZ) = ,3F10.2)
   05 FORMAT(14H (D ,O2,O1) = ,3F10.2)
      END

      SUBROUTINE PRINTF(F,NO,N1)
      DIMENSION F(800)
      DO 30 KO=1,NO
      KS=1+(KO-1)*(N1)
      KE=N1+(KO-1)*(N1)
      PRINT 11,KO
   30 PRINT 12,(F(KK),KK=KS,KE)
      RETURN
   11 FORMAT(11HOCURVE NO. ,I2)
   12 FORMAT(1H ,8F8.1)
      END

      SUBROUTINE SHIFT(F,NO,N1,CV,CS,CZ)
      DIMENSION F(800)
      P1=(F(1)+F(N1))/2.
      N3=(N1+1)/2
      P2=F(N3)
      DO 30 KO=1,NO
      KS=1+(KO-1)*N1
      KE=KO*N1
      KK=N3+(KO-1)*N1
      COR=(2.*P1+P2-F(KS)-F(KK)-F(KE))/3.
      DO 30 LO=1,N1
      KK=LO+(KO-1)*N1
   30 F(KK)=(F(KK)+COR)*CV/(CS-CZ)
      RETURN
      END

      SUBROUTINE SPCOOR(D,R)
      DIMENSION D(3),R(3)
      DATA PI/3.14159265358979/
      R(1)=SQRT(DOT(D,D))
      R(2)=0.
      R(3)=0.
      IF(R(1).EQ.0.) RETURN
      IF(D(1)**2+D(2)**2.NE.0.) R(2)=ATAN2(D(2),D(1))*180./PI
      R(3)=ATAN2(SQRT(D(1)**2+D(2)**2),D(3))*180./PI
      RETURN
      END

      FUNCTION DOT(X,Y)
      DIMENSION X(3),Y(3)
      DOT=X(1)*Y(1)+X(2)*Y(2)+X(3)*Y(3)
      RETURN
      END

      SUBROUTINE PRINTI(W)
      DIMENSION R(3),S(3),W(3)
      DATA ST,GT/1H-,1H+/
      PI=3.14159265
```

```
      DO 10 I=1,3                                                       SA201090
   10 S(I)=-W(I)                                                        SA201100
      CALL SPCOOR(S,R)                                                  SA201110
      W3=ABS(W(3))                                                      SA201120
      W32=.2*W3                                                         SA201130
      AM1=SQRT(S(1)**2+S(2)**2)                                         SA201140
      AM12=.2*AM1                                                       SA201150
      PRINT 11, AM1                                                     SA201160
      PRINT 12, R(2)                                                    SA201170
      PRINT 13, AM12                                                    SA201180
      PRINT 14, W3                                                      SA201190
      IF(S(3).LT.0.) PRINT 15, ST                                       SA201200
      IF(S(3).GE.0.) PRINT 15, GT                                       SA201210
      PRINT 16, W32                                                     SA201220
   11 FORMAT(54H      THE COMPENSATING MAGNET FOR THE XY-PLANE SHOULD B SA201230
     +,2HE ,F7.1,7H GAUSS-)                                             SA201240
   12 FORMAT(47H CENTIMETER-CUBED WITH THE NORTH POLE POINTING ,F6.1,   SA201250
     +17H DEGREES FROM +X.)                                             SA201260
   13 FORMAT(25H (THE MAGNET SHOULD READ ,F7.1,21H GAMMA AT ONE METER.))SA201270
   14 FORMAT(43H      THE COMPENSATING MAGNET FOR THE Z-AXIS,           SA201280
     +11H SHOULD BE ,F7.1,7H GAUSS-)                                    SA201290
   15 FORMAT(46H CENTIMETER-CUBED WITH THE NORTH POLE POINTING,         SA201300
     +9H TOWARDS ,A1,14HZ. (THE MAGNET)                                 SA201310
   16 FORMAT(13H SHOULD READ ,F7.1,22H GAMMA AT  ONE METER.))           SA201320
      RETURN                                                            SA201330
      END                                                               SA201340
```

APPENDIX D

SAMPLE PROBLEM FOR SA2024

```
NØL INTERCØM
TYPE "LØGIN."
LØGIN(S)
024533LACK/[redacted]/4


 09/07/73    09.46.40. BD/42/34
C- SETUP.FØRTRAN

ØN AT      09.46.56. 09/07/73
**FØRTRAN
**NEW ØR ØLD FILE- ATTACH(BN2024,BN2024)*ATTACH(DAT024,DAT024)*TAPE(ØN)

09.47.39.ATTACH(BN2024,BN2024)
09.47.50.ATTACH(DAT024,DAT024)
**READY.
BN2024.
     6    25      1
 96.00 1.000
     0
     6    25      0
 96.00 1.000
     1
    10     0 -5043 -4755 -3815 -1233  4384 12737 17492
 12596  4313 -1257 -3821 -4763 -5027 -4998 -4765 -4365
 -3863 -3410 -3226 -3394 -3865 -4359 -4767 -4990 -5037
 -9028 -8812 -8146 -6754 -4208 -1189   272 -1197 -4267
 -6727 -8155 -8806 -9041 -8976 -8510 -7287 -4838 -1645
    72 -1604 -4806 -7289 -8504 -8971 -9037-13025-12877
-12544-12066-11480-10968-10779-10971-11484-12066-12554
-12883-13030-12926-12190 -9905 -3946  5892 12194  5952
 -4064 -9930-12196-12910-13042-17035-16916-16423-15194  Data Tape
-12819 -9618 -8009 -9665-12770-15201-16425-16920-17041
-16906-16408-15151-12748 -9465 -7798 -9462-12745-15181
-16416-16911-17037-21024-20924-20199-17884-12022 -2110
  4183 -2072-12092-17925-20192-20911-21036-20884-20568
-20056-19474-18985-18775-18979-19477-20063-20545-20886
-21034-25024-24970-24520-23278-20863-17702-15987-17628
-20829-23299-24484-24970-25036-24817-24156-22743-20225
-17136-15667-17203-20207-22712-24146-24815-25035
     0     0     0
TAPE(ØFF)
```

```
ENTER 1 ØR 0 TØ INDICATE DATA PRINT ØR NØT:
(DX,DY,DZ) =       825.54    -211.43      20.71
(D ,02,01) =       852.44     -14.37      88.61
     THE CØMPENSATING MAGNET FØR THE XY-PLANE SHØULD BE    852.2 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING  165.6 DEGREES FRØM +X.
(THE MAGNET SHØULD READ    170.4 GAMMA AT ØNE METER.)
     THE CØMPENSATING MAGNET FØR THE Z-AXIS SHØULD BE     20.7 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING TØWARDS -Z. (THE MAGNET
SHØULD READ      4.1 GAMMA AT  ØNE METER.)

ENTER 1 ØR 0 TØ INDICATE DATA PRINT ØR NØT:

CURVE NO.  1
  -504.3  -475.5  -381.5  -123.3   438.4  1273.7  1749.2  1259.6
   431.3  -125.7  -382.1  -476.3  -502.7  -499.8  -476.5  -436.5
  -386.3  -341.0  -322.6  -339.4  -386.5  -435.9  -476.7  -499.0
  -503.7

CURVE NO.  2
  -502.8  -481.2  -414.6  -275.4   -20.8   281.1   427.2   280.3
   -26.7  -272.7  -415.5  -480.6  -504.1  -497.6  -451.0  -328.7
   -83.8   235.5   407.2   239.6   -80.6  -328.9  -450.4  -497.1
  -503.7

CURVE NO.  3
  -502.8  -488.0  -454.7  -406.9  -348.3  -297.1  -278.2  -297.4
  -348.7  -406.9  -455.7  -488.6  -503.3  -492.9  -419.3  -190.8
   405.1  1388.9  2019.1  1394.9   393.3  -193.3  -419.9  -491.3
  -504.5

CURVE NO.  4
  -503.3  -491.4  -442.1  -319.2   -81.7   238.4   399.3   233.7
   -76.8  -319.9  -442.3  -491.8  -503.9  -490.4  -440.6  -314.9
   -74.6   253.7   420.4   254.0   -74.3  -317.9  -441.4  -490.9
  -503.5

CURVE NO.  5
  -502.8  -492.8  -420.3  -188.8   397.4  1388.6  2017.9  1392.4
   390.4  -192.9  -419.6  -491.5  -504.0  -488.8  -457.2  -406.0
  -347.8  -298.9  -277.9  -298.3  -348.1  -406.7  -454.9  -489.0
  -503.8

CURVE NO.  6
  -502.8  -497.4  -452.4  -328.2   -86.7   229.4   400.9   236.8
   -83.3  -330.3  -448.8  -497.4  -504.0  -482.1  -416.0  -274.7
   -22.9   286.0   432.9   279.3   -21.1  -271.6  -415.0  -481.9
  -503.9
(DX,DY,DZ) =       825.54    -211.43      20.71
(D ,02,01) =       852.44     -14.37      88.61
     THE CØMPENSATING MAGNET FØR THE XY-PLANE SHØULD BE    852.2 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING  165.6 DEGREES FRØM +X.
(THE MAGNET SHØULD READ    170.4 GAMMA AT ØNE METER.)
```

```
     THE CØMPENSATING MAGNET FØR THE Z-AXIS SHØULD BE    20.7 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING TØWARDS -Z. (THE MAGNET
SHØULD READ      4.1 GAMMA AT  ØNE METER.)
09.53.46.STØP
**READY.
LØGØUT.

CP TIME      1.187
PP TIME     48.498
CØNNECT TIME  0 HR  9 MIN 10 SEC
TØTAL CØST ØF SESSIØN = $   2.28
 09/07/73  LØGGED ØUT AT 09.55.50.<
```

Notes:

1. The file BN2024 is the binary version of SA2024. It consists of six binary records (subprograms).

2. The information typed in by the user has been underlined.

APPENDIX E

LISTING OF SA3024

```
      PROGRAM DIPANL(INPUT=65,OUTPUT=65,DAT024=65,TAPE5=INPUT,
     + TAPE6=OUTPUT,TAPE7=DAT024,TAPE99)


C                          SATELLITE ANALYSIS PROGRAM


C       THIS PROGRAM ANALYZES MAGNETIC DATA REPRESENTING THE NORMAL COMPON-
C     ENT OF THE MAGNETIC FIELD FROM A SATELLITE. THE DATA IS ENTERED IN
C     THE FOLLOWING ORDER --

C     NO - THE NUMBER OF GREAT CIRCLES OF DATA. (NO IS USUALLY EVEN, E.G.,
C          NO=(N1-1)/2. THE PROGRAM STOPS IF NO=0.)

C     N1 - THE NUMBER OF DATA POINTS PER GREAT CIRCLE. (N1 IS ALWAYS ODD.
C          THE FIRST DATA POINT IS THE SAME AS THE LAST FOR EACH GREAT CIR-
C          LE.)

C     NH - THE HARMONIC NUMBER (DEGREE) REPRESENTING THE HIGHEST DEGREE
C          SPHERICAL HARMONIC TERM TO BE COMPUTED FROM THE DATA. (NH=1 FOR
C          DIPOLES, 2 FOR QUADRUPOLES, ETC.)

C     IR - DETERMINES WHETHER OR NOT TO READ THE DATA FROM THE FILE
C          DAT024. (IR=0 MEANS THAT THE DATA WILL NOT BE READ FROM
C          THE FILE.)

C     IP1 - DETERMINES WHETHER OR NOT THE MAGNETIC DATA IS TO BE PLOTTED.
C          (IP1=0 MEANS THAT THE DATA WILL NOT BE PLOTTED.)

C     IW - DETERMINES THE TYPE OF INTEGRATING SCHEME TO BE USED. (IW=0
C          MEANS THAT THE EXACT, ALGEBRAIC SCHEME IS TO BE USED.)

C     R1 - THE RADIUS OF THE MEASUREMENT SPHERE IN INCHES.

C     CV - THE VALUE OF THE CAL SIGNAL (CS-CZ) IN GAMMAS.

C     PY - THE SCALE FACTOR (GAMMAS/INCH) FOR THE Y-AXIS IF THE DATA IS TO
C          BE PLOTTED. (IF PY=0.0 A FACTOR WILL BE COMPUTED FROM THE DATA.)

C     IP2 - DETERMINES WHETHER OR NOT THE MAGNETIC DATA IS TO BE PRINTED.
C          (IP=0 MEANS THAT THE DATA WILL NOT BE PRINTED.)

C     CS - THE STATIC MEASUREMENT WITH THE CAL SIGNAL.

C     CZ - THE STATIC MEASUREMENT WITHOUT THE CAL SIGNAL.

C     F(I) - THE DATA REPRESENTING THE NORMAL COMPONENT OF THE MAGNETIC
C            FIELD ALONG GREAT CIRCLES ON THE MEASUREMENT SPHERE.
```

```
C ********************************************************************
      DIMENSION F(1000),PV(200),A(11),B(10),R(3),D(3)
      NAMELIST/NAM/NO,N1,NH,IR,IP1,IW,R1,CV,PY
      DATA PI/3.14159265358979/
      REWIND 7
      IPT=0
   10 READ(5,01) NO,N1,NH,IR,IP1,IW
      IF(IPT.EQ.0.AND.IP1.NE.0) CALL CALCM1(0,10H024 LACKEY,-10.)
      IF(NO.EQ.0) GO TO 90
      IF(IP1.NE.0) IPT=1
      READ(5,02) R1,CV,PY
      WRITE(6,NAM)
      WRITE(6,03)
      READ(5,01) IP2
      N3=NO*N1
      IF(IR.EQ.0) READ(5,04) CS,CZ,(F(I),I=1,N3)
      IF(IR.EQ.0) WRITE(7,04) CS,CZ,(F(I),I=1,N3)
      IF(IR.NE.0) READ(7,04) CS,CZ,(F(I),I=1,N3)
      CALL SHIFT(F,NO,N1,CV,CS,CZ)
      IF(IW.EQ.0) CALL WGT1(NO,N1,PV)
      IF(IW.NE.0) CALL WGT3(NO,N1,PV)
      CALL AMPMNT(NO,N1,0,R1,F,PV,A,B)
      A0=A(1)*1.E-5
      F0=A0/((R1*2.54)**2)
      DO 20 I=1,N3
   20 F(I)=F(I)*1.E-5-F0
      WRITE(6,05) A0
      IF(IP2.NE.0) CALL PRINTF(F,NO,N1)
      IF(IP1.NE.0) CALL DATPLT(NO,N1,F,PY)
      DO 80 NI=1,NH
      CALL AMPMNT(NO,N1,NI,R1,F,PV,A,B)
      IF(NI.GT.1) GO TO 60
      D(1)=A(2)
      D(2)=B(1)
      D(3)=A(1)
      CALL SPCOOR(D,R)
      WRITE(6,06) D
      WRITE(6,07) R
      CALL PRINTI(D)
      GO TO 70
   60 IF(NI.GT.2) GO TO 70
      S3=SQRT(3.)
      Q11=S3*A(3)-A(1)
      Q22=-S3*A(3)-A(1)
      Q33=-Q11-Q22
      Q12=S3*B(2)
      Q13=S3*A(2)
      Q23=S3*B(1)
      WRITE(6,09) Q11,Q22,Q33,Q12,Q13,Q23
   70 WRITE(6,11) NI
      NI1=NI+1
      WRITE(6,12) (A(I),I=1,NI1)
      WRITE(6,12) (B(I),I=1,NI)
   80 CONTINUE
      GO TO 10
   90 IF(IPT.NE.0) CALL CALCM1(0,10H024 LACKEY,+10.)
      STOP
   01 FORMAT(14I5)
```

```
02 FORMAT(9F8.4)
03 FORMAT(44H0ENTER 1 OR 0 TO INDICATE DATA PRINT OR NOT9)
04 FORMAT(9F6.1)
06 FORMAT(14H (DX,DY,DZ) = ,3F10.2)
05 FORMAT(26H0THE MONOPOLE MOMENT IS --,F20.6//)
07 FORMAT(14H (D ,02,01) = ,3F10.2)
09 FORMAT(54H0THE QUADRUPOLE MOMENTS Q11,Q22,Q33,Q12,Q13,Q23 ARE --/
  + (3E20.12))
11 FORMAT(27H0THE A ( B COEFFS. FOR THE ,I2,16HTH HARMONIC ARE9)
12 FORMAT(1H ,5E20.12)
   END

   SUBROUTINE AMPMNT(NO,N1,N,R1,F,P,A,B)
   DIMENSION F(1000),P(200),PN(10,200),A(11),B(10)
   DATA PI/3.14159265358979323846/
   CALL POLVAL(N1,N,PN)
   KL=0
   N2=(N1+1)/2
   D1=PI/FLOAT(NO)
   D2=2.*PI/FLOAT(N1-1)
   DO 10 K=1,NO
   A1=D1*FLOAT(K-1)
   LP=+1
   LI=0
   DO 10 L=1,N1
   KL=KL+1
   IF(L.GT.N2) LP=-1
   LI=LI+LP
   A2=A1+FLOAT(1-LP)*PI/2.
   F1=F(KL)
   A2N=FLOAT(2*N+1)
   A1N=FLOAT(N+1)
   AR=(R1*2.54)**(N+2)
   NN=N+1
   DO 10 MM=1,NN
   M=MM-1
   AM=FLOAT(M)
   IF(KL.EQ.1) A(MM)=0.
   IF(KL.EQ.1.AND.M.GT.0) B(M)=0.
   PNM1=PN(MM,LI)*COS(AM*A2)
   PNM2=PN(MM,LI)*SIN(AM*A2)
   AC=(A2N*AR/A1N)*F1*P(LI)
   A(MM)=A(MM)+AC*PNM1
   IF(M.GT.0) B(M)=B(M)+AC*PNM2
10 CONTINUE
   RETURN
   END

   SUBROUTINE WGT1(NO,N1,P)
   DIMENSION D(50),P(200)
   N4=(N1+1)/4
   AN=FLOAT(NO)
   CALL WGT2(N1,D)
   DO 10 J1=1,N4
   J3=N1/2+2-J1
   DJ=D(J1)/(4.*AN)
   P(J1)=DJ
10 P(J3)=DJ
   IF(INT(FLOAT(N1-1)/4.+.1)*4.NE.(N1-1)) GO TO 20
   J1=N4+1
```

```
      P(J1)=D(J1)/(4.*AN)
   20 N4=(N1+1)/2
      P(N4)=P(N4)*2.
      RETURN
      END

      SUBROUTINE WGT2(N,D)
      DIMENSION A(50,50),D(50),C(50)
      DOUBLE PRECISION A,C,PI,AN
      DATA PI/3.14159265358979323846264300D0/
      N3=(N+1)/4
      N4=N3+1
      DO 10 I=2,N4
      C(I)=1./FLOAT(2*I-1)
      A(I,1)=1.
      A(I,N4)=0.
      DO 10 J=2,N3
      AN=2.*PI*DBLE(FLOAT(J-1))/DBLE(FLOAT(N-1))
   10 A(I,J)=(DCOS(AN))**(2*I-2)
      C(1)=2.
      A(1,N4)=1.
      DO 20 J=1,N3
   20 A(1,J)=2.
      N3=N4
      IF(INT(FLOAT(N-1)/4.+.1)*4.NE.(N-1)) N3=N3-1
      CALL GAUSEL(A,C,N3,N3,1)
      DO 30 I=1,N3
   30 D(I)=C(I)
      RETURN
      END

      SUBROUTINE WGT3(NO,N1,P)
      DIMENSION P(200)
      DATA PI/3.14159265358979323846/
      D2=2.*PI/FLOAT(N1-1)
      E1=(SIN(D2/4.))**2
      E2=SIN(D2/2.)
      P(1)=E1/(2.*FLOAT(NO))
      N2=(N1+1)/2
      P(N2)=P(1)*2.
      IF(N1.LE.3) RETURN
      N3=N2-1
      DO 10 L=2,N3
      S2=SIN(D2*FLOAT(L-1))
      P(L)=ABS(S2*E2)/(2.*FLOAT(NO))
   10 CONTINUE
      RETURN
      END

      SUBROUTINE GAUSEL(A,C,M,N,IT)
      DIMENSION A(50,50),C(50),IB(50)
      DOUBLE PRECISION A,C,D
C     WRITE(6,03)
C     DO 20 I=1,M
C  20 WRITE(6,04) (A(I,J),J=1,N),C(I)
      DO 70 I=1,N
   70 IB(I)=I
      DO 60 I=1,M
      IF(IT.NE.0.AND.M.NE.N) GO TO 35
      D=0.
```

```
      JJ=0                                                              SA302290
      DO 10 J=I,N                                                       SA302300
      IF(DABS(A(I,J)).LE.D) GO TO 10                                    SA302310
      JJ=J                                                              SA302320
      D=DABS(A(I,J))                                                    SA302330
   10 CONTINUE                                                          SA302340
      IF(JJ.EQ.I) GO TO 35                                              SA302350
      IF(JJ.NE.0) GO TO 140                                             SA302360
      WRITE(6,01) I                                                     SA302370
      GO TO 110                                                         SA302380
  140 DO 160 J=1,M                                                      SA302390
      D=A(J,JJ)                                                         SA302400
      A(J,JJ)=A(J,I)                                                    SA302410
  160 A(J,I)=D                                                          SA302420
      ID=IB(JJ)                                                         SA302430
      IB(JJ)=IB(I)                                                      SA302440
      IB(I)=ID                                                          SA302450
   35 D=A(I,I)                                                          SA302460
      DO 30 J=I,M                                                       SA302470
   30 A(I,J)=A(I,J)/D                                                   SA302480
      C(I)=C(I)/D                                                       SA302490
      DO 50 J=1,M                                                       SA302500
      IF(J.EQ.I) GO TO 50                                               SA302510
      D=A(J,I)                                                          SA302520
      IF(D.EQ.0.) GO TO 50                                              SA302530
      DO 40 K=I,N                                                       SA302540
   40 A(J,K)=A(J,K)-D*A(I,K)                                            SA302550
      C(J)=C(J)-D*C(I)                                                  SA302560
   50 CONTINUE                                                          SA302570
   60 CONTINUE                                                          SA302580
  110 IF(IT.EQ.0.OR.M.NE.N) GO TO 100                                   SA302590
      DO 120 I=1,N                                                      SA302600
  120 A(I,N)=C(I)                                                       SA302610
      DO 130 I=1,N                                                      SA302620
      II=IB(I)                                                          SA302630
  130 C(II)=A(I,N)                                                      SA302640
  100 CONTINUE                                                          SA302650
C     IF(IT.EQ.0) WRITE(6,02) (IB(I),I=1,N)                             SA302660
C     WRITE(6,05)                                                       SA302670
C     DO 80 I=1,M                                                       SA302680
C  80 WRITE(6,04) (A(I,J),J=1,N),C(I)                                   SA302690
      RETURN                                                            SA302700
   01 FORMAT(5H1ROW ,I2,14H IS ALL ZEROS.)                              SA302710
   02 FORMAT(24H0SINGLE PERMUTATION IS...,20I3//)                       SA302720
   03 FORMAT(21H0THE INPUT MATRIX IS9//)                                SA302730
   04 FORMAT(1H ,7D16.8)                                                SA302740
   05 FORMAT(22H0THE OUTPUT MATRIX IS9//)                               SA302750
      END                                                               SA302760
                                                                        SA302770
      SUBROUTINE POLVAL(N1,N,PN)                                        SA302780
      DIMENSION PN(10,200)                                              SA302790
      DATA PI/3.14159265358979323846/                                   SA302800
      D2=2.*PI/FLOAT(N1-1)                                              SA302810
      N2=(N1+1)/2                                                       SA302820
      DO 10 I=1,N2                                                      SA302830
      A2=D2*FLOAT(I-1)                                                  SA302840
      C2=COS(A2)                                                        SA302850
      NN=N+1                                                            SA302860
      DO 10 MM=1,NN                                                     SA302870
      M=MM-1                                                            SA302880
```

```
      PN(MM,I)=SPNM(N,M,C2)
   10 CONTINUE
      RETURN
      END

      SUBROUTINE SHIFT(F,NO,N1,CV,CS,CZ)
      DIMENSION F(800)
      P1=(F(1)+F(N1))/2.
      N3=(N1+1)/2
      P2=F(N3)
      DO 10 KO=1,NO
      KS=1+(KO-1)*N1
      KE=KO*N1
      KK=N3+(KO-1)*N1
      COR=(2.*P1+P2-F(KS)-F(KK)-F(KE))/3.
      DO 10 LO=1,N1
      KK=LO+(KO-1)*N1
   10 F(KK)=(F(KK)+COR)*CV/(CS-CZ)
      RETURN
      END

      SUBROUTINE PRINTI(W)
      DIMENSION R(3),S(3),W(3)
      DATA ST,GT/1H-,1H+/
      P1=3.14159265
      DO 10 I=1,3
   10 S(I)=-W(I)
      R(1)=SQRT(S(1)**2+S(2)**2+S(3)**2)
      R(2)=ATAN2(S(2),S(1))*180./P1
      R(3)=ATAN2(SQRT(S(1)**2+S(2)**2),S(3))*180./P1
      W3=ABS(W(3))
      W32=.2*W3
      AM1=SQRT(S(1)**2+S(2)**2)
      AM12=.2*AM1
      PRINT 11, AM1
      PRINT 12, R(2)
      PRINT 13, AM12
      PRINT 14, W3
      IF(S(3).LT.0.) PRINT 15, ST
      IF(S(3).GE.0.) PRINT 15, GT
      PRINT 16, W32
   11 FORMAT(54H      THE COMPENSATING MAGNET FOR THE XY-PLANE SHOULD B
     +,2HE ,F7.1,7H GAUSS-)
   12 FORMAT(47H CENTIMETER-CUBED WITH THE NORTH POLE POINTING ,F6.1,
     +17H DEGREES FROM +X.)
   13 FORMAT(25H (THE MAGNET SHOULD READ ,F7.1,21H GAMMA AT ONE METER.))
   14 FORMAT(43H      THE COMPENSATING MAGNET FOR THE Z-AXIS,
     +11H SHOULD BE ,F7.1,7H GAUSS-)
   15 FORMAT(46H CENTIMETER-CUBED WITH THE NORTH POLE POINTING,
     +9H TOWARDS ,A1,14HZ. (THE MAGNET)
   16 FORMAT(13H SHOULD READ ,F7.1,22H GAMMA AT  ONE METER.))
      RETURN
      END

      FUNCTION SPNM(N,M,X)
      SPNM=PNM(N,M,X)
      IF(M.EQ.0.OR.M.GT.N) RETURN
      SPNM=SQRT(2.*ANF(N-M)/ANF(N+M))*SPNM
      RETURN
      END
```

```
      FUNCTION PNM(N,M,X)
      PNM=0.
      IF(M.GT.N) RETURN
      IF((ABS(1.-ABS(X))).GT.1.E-9) GO TO 20
      IF(M.NE.0) RETURN
      PNM=-1.
      IF(X.GT.0..OR.N.EQ.INT(FLOAT(N)/2.+.1)*2) PNM=1.
      RETURN
   20 CNM=(2.**N)*ANF(N)*ANF(N-M)
      PNM=1.
      IF(M.NE.0) PNM=SQRT(1.-X*X)**M
      CNM=ANF(2*N)*PNM/CNM
      PNM=1.
      IF(M.NE.N) PNM=X**(N-M)
      IF(N-M.LE.1) GO TO 40
      PRD1=1.
      NT=(N-M)/2
      DO 30 I=1,NT
      AN1=FLOAT(N-M-2*I+2)
      AN2=FLOAT(2*I)
      AN3=FLOAT(2*N-2*I+1)
      PRD1=-PRD1*AN1*(AN1-1)/(AN2*AN3)
      NE=N-M-2*I
      AN1=1.
      IF(NE.GT.0) AN1=X**NE
      PNM=PNM+PRD1*AN1
   30 CONTINUE
   40 PNM=CNM*PNM
      RETURN
      END

      FUNCTION ANF(N)
      DOUBLE PRECISION AN
      ANF=1.
      AN=1.D0
      IF(N.LT.0) PRINT 01
      IF(N.LT.2) RETURN
      DO 10 I=2,N
   10 AN=AN*DBLE(FLOAT(I))
      ANF=SNGL(AN)
      RETURN
   01 FORMAT(37H1FACTORIAL INTEGER IS LESS THAN ZERO.//)
      END

      SUBROUTINE SPCOOR(D,R)
      DIMENSION D(3),R(3)
      DATA PI/3.14159265358979/
      R(1)=SQRT(DOT(D,'
      R(2)=0.
      R(3)=0.
      IF(R(1).EQ.0.) RETURN
      IF(D(1)**2+D(2)**2.NE.0.) R(2)=ATAN2(D(2),D(1))*180./PI
      R(3)=ATAN2(SQRT(D(1)**2+D(2)**2),D(3))*180./PI
      RETURN
      END

      SUBROUTINE SUM(A,X,B,Y,Z)
      DIMENSION X(3),Y(3),Z(3)
      DO 10 I=1,3
```

```
10 Z(I)=A*X(I)+B*Y(I)
   RETURN
   END

   FUNCTION DOT(X,Y)
   DIMENSION X(3),Y(3)
   DOT=X(1)*Y(1)+X(2)*Y(2)+X(3)*Y(3)
   RETURN
   END

   SUBROUTINE PRINTF(F,NO,N1)
   DIMENSION F(800),F1(100)
   DO 20 KO=1,NO
   DO 10 I=1,N1
   II=I+(KO-1)*N1
10 F1(I)=F(II)*1.E5
   WRITE(6,01) KO
20 WRITE(6,02) (F1(KK),KK=1,N1)
01 FORMAT(11HOCURVE NO. ,I2)
02 FORMAT(1H ,8F8.1)
   RETURN
   END

   SUBROUTINE DATPLT(NO,N1,F,YDIST)
   DIMENSION F(1000),F1(33),F2(73),X(33),X1(73),C(1 ),P(1 ),CH(16)
   DATA PI/3.14159265358979/
   DATA CH/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13,
  + 2H14,2H15,2H16/
   N3=NO*N1
   CALL YSCALE(N3,F,YDIST,YMAX,YMIN)
   DX=360./FLOAT(N1-1)
   F1(1)=F1(2)=YMAX $ X(1)=0. $ X(2)=360.
   CALL CALCM1(2,X,F1,0,0.,360.,YMIN,YMAX,6.,7.,TITLE,0,
  + 20HCOLATITUDE (DEGREES),-20,30HNORMAL MAGNETIC FIELD (GAMMAS),
  + 30,0.,18)
   F1(2)=YMIN $ X(1)=X(2)=360.
   CALL CALCM1(-2,X,F1,0)
   F1(1)=F1(2)=0.    $ X(1)=360. $ X(2)=0.
   IF(YMAX.GT.0.0.AND.YMIN.LT.0.0) CALL CALCM1(-2,X,F1,0)
   DO 10 I=1,N1
10 X(I)=DX*FLOAT(I-1)
   DO 20 I=1,73
20 X1(I)=5.*FLOAT(I-1)
   IJ=0
   DO 50 I=1,NO
   DO 30 J=1,N1
   IJ=IJ+1
30 F1(J)=F(IJ)*1.E+5
   CALL FNCTON(F1,N1-1,C,CO,P,N2,DEV)
   DO 40 N=1,N2
   EN=N
   DO 40 J=1,73
   XX=X1(J)*PI/180.
   IF(N.EQ.1) F2(J)=CO
40 F2(J)=F2(J)+C(N)*SIN(EN*XX+P(N))
   CALL CALCM1(-73,X1,F2,0)
   DO 50 J=1,N1
   NC=1+I/10
   CALL SYMBL4(X(J)/60.,(F1(J)-YMIN)/YDIST,.08,CH(I),0.,NC)
50 CONTINUE
```

```
      RETURN
      END

      SUBROUTINE FNCTON(F,NSCANS,C,CO,PHI,N2,DEV)
      DIMENSION F(1),G(33),C(1),PHI(1)
      DATA PI/3.14159265358979/
      DX=2.*PI/FLOAT(NSCANS)
      NE=NSCANS/2+1
      CO=0.
      DO 10 J=1,NSCANS
   10 CO=CO+F(J)/FLOAT(NSCANS)
      DO 30 N=1,NE
      EN=N
      S1=0.
      C1=0.
      X=-DX
      DO 20 J=1,NSCANS
      X=X+DX
      S1=S1+(1./PI)*F(J)*SIN(EN*X)*DX
   20 C1=C1+(1./PI)*F(J)*COS(EN*X)*DX
      C(N)=SQRT(S1**2+C1**2)
      IF((FLOAT(N)).EQ.(FLOAT(NSCANS)/2.)) C(N)=C(N)/2.
      PHI(N)=0.0
   30 IF((S1**2+C1**2).NE.0.) PHI(N)=ATAN2(C1,S1)
      DO 50 N=1,NE
      EN=N
      X=-DX
      DEV1=0.
      DO 40 J=1,NSCANS
      X=X+DX
      IF(N.EQ.1) G(J)=CO
      G(J)=G(J)+C(N)*SIN(EN*X+PHI(N))
   40 IF(ABS(G(J)-F(J)).GT.DEV1) DEV1=ABS(G(J)-F(J))
      IF(N.EQ.1) DEV=DEV1
      IF(N.EQ.1) N2=1
      IF(DEV1.LT.DEV) N2=N
      IF(DEV1.LT.DEV) DEV=DEV1
   50 CONTINUE
      RETURN
      END

      SUBROUTINE YSCALE(N,F,YD,YX,YM)
      DIMENSION F(800)
      PP=F(1)
      PN=PP
      DO 10 I=1,N
      IF(F(I).GT.PP) PP=F(I)
      IF(F(I).LT.PN) PN=F(I)
   10 CONTINUE
      PP=PP*1.E+5
      PN=PN*1.E+5
      P=PP-PN
      SN=0.
      IF(PP.GT.0.) SN=1.
      IF(YD.NE.0.) GO TO 50
      DO 20 I=1,21
      FA=(1.E-9)*(10.**I)
      FA1=FA*.1
      IF(P/FA.GE.1.) GO TO 20
      GO TO 30
```

```
20 CONTINUE                                                             SA305290
30 YD=5.*FA1                                                            SA305300
   YX=FLOAT(INT(PP/YD+SN))*YD                                           SA305310
   YM=YX-7.*YD                                                          SA305320
   CN=4.                                                                SA305330
   DO 40 I=1,3                                                          SA305340
   CN=CN*.5                                                             SA305350
   YD1=CN*FA1                                                           SA305360
   YX1=FLOAT(INT(PP/YD1+SN))*YD1                                        SA305370
   YM1=YX1-7.*YD1                                                       SA305380
   IF((7.*YD1-P).LT.0.0.OR.YX1.LT.PP.OR.YM1.GT.PN) RETURN               SA305390
   YD=YD1                                                               SA305400
   YX=YX1                                                               SA305410
   YM=YM1                                                               SA305420
40 CONTINUE                                                             SA305430
50 YX=FLOAT(INT(PP/YD+SN))*YD                                           SA305440
   YM=YX-7.*YD                                                          SA305450
   RETURN                                                               SA305460
   END                                                                  SA305470
```

APPENDIX F

SAMPLE PROBLEMS FOR SA3024

I. PROBLEM EXECUTED ON INTERCOM (TIME-SHARING SYSTEM)

```
NØL INTERCØM
TYPE "LØGIN."
LØGIN(S)
024533LACK/[redacted]/4

 09/07/73    14.35.39. BD/42/31
C- SETUP.FØRTRAN

ØN AT      14.36.03. 09/07/73
**FØRTRAN
**NEW ØR ØLD FILE- ATTACH(BN3024,BN3024)*ATTACH(DAT024,DAT024)

14.36.38.ATTACH(BN3024,BN3024)
14.36.42.ATTACH(DAT024,DAT024)
**READY.
REWIND(BN3024)*CØPYBR(BN3024,FIL,16)*RETURN(BN3024)*TAPE(ØN)

**READY.
FIL.
    6   25     3     1     0     1
   96.00    1.000    0.000
    1
    6   25     3     0     0     0
   96.00    1.000    0.000
    0
    10     0 -5043 -4755 -3815 -1233  4384 12737 17492
 12596  4313 -1257 -3821 -4763 -5027 -4998 -4765 -4365
 -3863 -3410 -3226 -3394 -3865 -4359 -4767 -4990 -5037
 -9028 -8812 -8146 -6754 -4208 -1189   272 -1197 -4267
 -6727 -8155 -8806 -9041 -8976 -8510 -7287 -4838 -1645
    72 -1604 -4806 -7289 -8504 -8971 -9037-13025-12877
-12544-12066-11480-10968-10779-10971-11484-12066-12554
-12883-13030-12926-12190 -9905 -3946  5892 12194  5952   Data Tape
 -4064 -9930-12196-12910-13042-17035-16916-16423-15194
-12819 -9618 -8009 -9665-12770-15201-16425-16920-17041
-16906-16408-15151-12748 -9465 -7798 -9462-12745-15181
-16416-16911-17037-21024-20924-20199-17884-12022 -2110
  4183 -2072-12092-17925-20192-20911-21036-20884-20568
-20056-19474-18985-18775-18979-19477-20063-20545-20886
-21034-25024-24970-24520-23278-20863-17702-15987-17628
```

```
-20829-23299-24484-24970-25036-24817-24156-22743-20225   Data Tape
-17136-15667-17203-20207-22712-24146-24815-25035
    0      0      0      0      0      0
TAPE(ØFF)
```

```
$NAM

NO       =   6,

N1       =   25,

NH       =   3,

IR       =   1,

IP1      =   0,

IW       =   1,

R1       =   0.96E+02,

CV       =   0.1E+01,

PY       =   0.0,

$END

ENTER 1 ØR 0 TØ INDICATE DATA PRINT ØR NØT:

THE MØNØPØLE MØMENT IS --          14.339536


CURVE NO.   1
   -528.4   -499.6   -405.6   -147.4    414.3   1249.6   1725.1   1235.5
    407.2   -149.8   -406.2   -500.4   -526.8   -523.9   -500.6   -460.6
   -410.4   -365.1   -346.7   -363.5   -410.6   -460.0   -500.8   -523.1
   -527.8

CURVE NO.   2
   -527.0   -505.4   -438.8   -299.6    -45.0    256.9    403.0    256.1
    -50.9   -296.9   -439.7   -504.8   -528.3   -521.8   -475.2   -352.9
   -108.0    211.3    383.0    215.4   -104.8   -353.1   -474.6   -521.3
   -527.9
```

```
CURVE NO.  3
   -527.0  -512.2  -478.9  -431.1  -372.5  -321.3  -302.4  -321.6
   -372.9  -431.1  -479.9  -512.8  -527.5  -517.1  -443.5  -215.0
    380.9  1364.7  1994.9  1370.7   369.1  -217.5  -444.1  -515.5
   -528.7

CURVE NO.  4
   -527.4  -515.5  -466.2  -343.3  -105.8   214.3   375.2   209.6
   -100.9  -344.0  -466.4  -515.9  -528.0  -514.5  -464.7  -339.0
    -98.7   229.6   396.3   229.9   -96.4  -342.0  -465.5  -515.0
   -527.6

CURVE NO.  5
   -527.0  -517.0  -444.5  -213.0   373.2  1364.4  1993.7  1368.2
    366.2  -217.1  -443.8  -515.7  -528.2  -513.0  -481.4  -430.2
   -372.0  -323.1  -302.1  -322.5  -372.3  -430.9  -479.1  -513.2
   -528.0

CURVE NO.  6
   -526.9  -521.5  -476.5  -352.3  -110.8   205.3   376.8   212.7
   -107.4  -354.4  -472.9  -521.5  -528.1  -506.2  -440.1  -298.8
    -47.0   261.9   408.8   255.2   -45.2  -295.7  -439.1  -506.0
   -528.0
(DX,DY,DZ) =      825.54   -211.43     20.71
(D ,Ø2,Ø1) =      852.44    -14.37     88.61
     THE CØMPENSATING MAGNET FØR THE XY-PLANE SHØULD BE   852.2 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING  165.6 DEGREES FRØM +X.
(THE MAGNET SHØULD READ   170.4 GAMMA AT ØNE METER.)
     THE CØMPENSATING MAGNET FØR THE Z-AXIS SHØULD BE    20.7 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING TØWARDS -Z. (THE MAGNET
SHØULD READ     4.1 GAMMA AT  ØNE METER.)

THE A & B CØEFFS. FØR THE  1TH HARMØNIC ARE:
    .207106285746E+02   .825542024056E+03
   -.211426879295E+03

THE QUADRUPØLE MØMENTS Q11,Q22,Q33,Q12,Q13,Q23 ARE --
   .913801584473E+07   .996620668908E+07  -.191042225338E+08
   .141965058911E+05   .189439835558E+05   .902315837020E+04

THE A & B CØEFFS. FØR THE  2TH HARMØNIC ARE:
   -.955211126691E+07   .109373140054E+05  -.239078103461E+06
    .520952291397E+04   .819635649778E+04

THE A & B CØEFFS. FØR THE  3TH HARMØNIC ARE:
   -.108665709727E+07   .734751632135E+08   .261662613936E+07   .23785770
1231E+10
    .446316961324E+07   .306933172460E+06   .393914212069E+07
```

```
$NAM

NO       =  6,

N1       =  25,

NH       =  3,

IR       =  0,

IP1      =  0,

IW       =  0,

R1       =  0.96E+02,

CV       =  0.1E+01,

PY       =  0.0,

$END

ENTER 1 ØR 0 TØ INDICATE DATA PRINT ØR NØT:

THE MØNØPØLE MØMENT IS --            15.233732


(DX,DY,DZ) =      827.69    -212.04     20.70
(D ,Ø2,Ø1) =      854.67    -14.37      88.61
    THE CØMPENSATING MAGNET FØR THE XY-PLANE SHØULD BE   854.4 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING  165.6 DEGREES FRØM +X.
(THE MAGNET SHØULD READ   170.9 GAMMA AT ØNE METER.)
    THE CØMPENSATING MAGNET FØR THE Z-AXIS SHØULD BE    20.7 GAUSS-
CENTIMETER-CUBED WITH THE NØRTH PØLE PØINTING TØWARDS -Z. (THE MAGNET
SHØULD READ     4.1 GAMMA AT  ØNE METER.)

THE A & B CØEFFS. FØR THE  1TH HARMØNIC ARE:
   .206988483703E+02   .827692992075E+03
  -.212041720726E+03

THE QUADRUPØLE MØMENTS Q11,Q22,Q33,Q12,Q13,Q23 ARE --
  .907751557573E+07   .990809608352E+07  -.189856116592E+08
  .142413793441E+05   .190259234406E+05   .904881389053E+04

THE A & B CØEFFS. FØR THE  2TH HARMØNIC ARE:
  -.949280582962E+07   .109846220200E+05  -.239767939880E+06
   .522433513554E+04   .822226419797E+04

THE A & B CØEFFS. FØR THE  3TH HARMØNIC ARE:
  -.109329743425E+07   .736637847141E+08   .262663903488E+07   .23853972
2966E+10
   .447595915074E+07   .307504565712E+06   .394909710659E+07
14.47.58.STØP
**READY.
```

```
LØGØUT.

CP TIME      2.824
PP TIME     80.985
CØNNECT TIME  0 HR 14 MIN 12 SEC
TØTAL CØST ØF SESSIØN = $    3.72
 09/07/73  LØGGED ØUT AT 14.49.51.<
```

Notes:

1. The file BN3024 is the binary version of SA3024. It consists of 19 binary records (subprograms). The last three are plotting routines and are not used when executing problems on INTERCOM.

2. The information typed in by the user has been underlined.

APPENDIX (CONT.)

II. PROBLEM SUBMITTED TO BATCH

```
NØL INTERCØM
TYPE "LØGIN."
LØGIN(S)
024533LACK/[redacted]/4

 09/07/73   14.50.40. BD/42/31
C- SETUP.GENERAL

ØN AT      14.51.17. 09/07/73
**GENERAL
**NEW ØR ØLD FILE- NEW/IECC024*TAPE(ØN)

**READY.
1IECC3ST,F1,T060,CM060000.55302435,024,LACKEY.
2ATTACH(ABC,NØLBIN)
3CØPYN(0,DEF,ABC)
4RETURN(ABC)
5ATTACH(MHL,DAT024)
6REWIND(MHL)
7CØPYBF(MHL,DAT024)
8RETURN(MHL)
9ATTACH(BN3024,BN3024)
10LØAD(BN3024)
11DEF.
12*WEØR
14REWIND(ABC)
15GØULD1,14,ABC
16*WEØR
5490     6    25     3     1     1     1
5500    96.00    1.000    0.000
5510     1
5520     6    25     3     0     1     0
5530    96.00    1.000    0.000
5540     0
5550    10      0 -5043 -4755 -3815 -1233  4384 12737 17492
5560 12596  4313 -1257 -3821 -4763 -5027 -4998 -4765 -4365
5570 -3863 -3410 -3226 -3394 -3865 -4359 -4767 -4990 -5037
5580 -9028 -8812 -8146 -6754 -4208 -1189   272 -1197 -4267
5590 -6727 -8155 -8806 -9041 -8976 -8510 -7287 -4838 -1645
5600    72 -1604 -4806 -7289 -8504 -8971 -9037-13025-12877
5610-12544-12066-11480-10968-10779-10971-11484-12066-12554
```

Tape prepared beforehand in LOCAL mode

```
5620-12883-13030-12926-12190 -9905 -3946  5892 12194  5952
5630 -4064 -9930-12196-12910-13042-17035-16916-16423-15194
5640-12819 -9618 -8009 -9665-12770-15201-16425-16920-17041
5650-16906-16408-15151-12748 -9465 -7798 -9462-12745-15181
5660-16416-16911-17037-21024-20924-20199-17884-12022 -2110
5670  4183 -2072-12092-17925-20192-20911-21036-20884-20568
5680-20056-19474-18985-18775-18979-19477-20063-20545-20886
5690-21034-25024-24970-24520-23278-20863-17702-15987-17628
5700-20829-23299-24484-24970-25036-24817-24156-22743-20225
5710-17136-15667-17203-20207-22712-24146-24815-25035
5720     0     0     0     0     0     0
TAPE(ØFF)
```

Tape prepared beforehand in LOCAL mode

```
SAVE*PURGE(IECC024)*BATCH.*QUEUES.

**SAVED IECC024
14.58.13.PURGE(IECC024)
TYPE FILE NAME-IECC024

TYPE DISPØSITIØN-INPUT

TYPE FILE NAME-END


QUEUES  15.00.35. I= 19, Ø=   3, P=   1, C=   2.
INPUT = 19
CBCGU81-5 ECAAG8Y-3 HHJFI8B-2 SAFCS8N-2 DCCXX8Z-3 SAFCS8I-2
BCAJH8E-2 HHJ1J74-2 HHJFI8A-2 AJFG084-3 SAFHF71-2 IECC386-1
ICCBG8X-4 IGBJE7T-2 HHJMV8M-2 HHJ1J8L-2 DBCJG8W-2 IAJFS83-5
DCAEV8U-5
ØUTPUT=  3
BDASH80-0 IAJF03P-0 DAYFI7G-0
PUNCH =  1
BDASH80-0
CØMMØN=  2
SSSSSSU-0 SSSSSST-0
CØNTRØL PTS.
AJFVD78-2 HHJFI79-5 GRID68P-5 AJFG06Q-4 AJFXF54-2 GRIDF80-5
IAJF082-5
15.00.35.STØP
**READY.
LØGØUT.

CP TIME      .949
PP TIME   144.517
CONNECT TIME  0 HR 11 MIN  2 SEC
TØTAL CØST ØF SESSIØN = $    3.75
 09/07/73  LØGGED ØUT AT 15.01.42.<
```

Note:

1. The information typed in by the user has been underlined.

APPENDIX G

LISTING OF SA4024

```
      PROGRAM DIPDAT(INPUT=65,OUTPUT=65,TAPE5=INPUT,TAPE6=OUTPUT,
     + DAT024=65,TAPE7=DAT024,TAPE99)


C                         SATELLITE DATA PROGRAM


C       THIS PROGRAM GENERATES AND PLOTS DATA REPRESENTING THE NORMAL COM-
C     PONENT OF THE MAGNETIC FIELD FROM A SATELLITE. THE DATA IS ENTERED
C     IN THE FOLLOWING ORDER --

C     NO - THE NUMBER OF GREAT CIRCLES OF DATA. (NO IS USUALLY EVEN, E.G.,
C          NO=(N1-1)/2. THE PROGRAM STOPS IF NO=0.)

C     N1 - THE NUMBER OF DATA POINTS PER GREAT CIRCLE. (N1 IS ALWAYS ODD.
C          THE FIRST DATA POINT IS THE SAME AS THE LAST FOR EACH GREAT CIR-
C          LE.)

C     NH - THE TOTAL NUMBER OF DIFFERENT HARMONICS (DEGREES) OF MULTIPOLE
C          MAGNETS TO BE CONSIDERED.

C     IP - DETERMINES WHETHER OR NOT THE MAGNETIC DATA IS TO BE PLOTTED.
C          (IP=0 MEANS THAT THE DATA WILL NOT BE PLOTTED.)

C     R1 - THE RADIUS OF THE MEASUREMENT SPHERE IN INCHES.

C     EG - THE ERROR (IN GAMMAS) TO BE RANDOMLY INSERTED INTO THE DATA TO
C          REPRESENT INSTRUMENTATION INACCURACIES.

C     EA - THE ERROR (IN DEGREES) TO BE RANDOMLY INSERTED INTO THE DATA TO
C          REPRESENT MEASUREMENT POSITION ERRORS.

C     ED - THE CONSTANT ERROR (IN GAMMAS) TO BE INSERTED INTO THE DATA.
C          (THIS WILL BE ANALYZED AS MONOPOLE MOMENT.)

C     PY - THE SCALE FACTOR (GAMMAS/INCH) FOR THE Y-AXIS IF THE DATA IS TO
C          BE PLOTTED. (IF PY=0.0 A FACTOR WILL BE COMPUTED FROM THE DATA.)

C     F9 - THE FORMAT FOR READING AND PRINTING THE SPHERICAL COEFFICIENTS
C          A(I) AND B(I).



C     DO ** NI=1,NH

C       NN - THE HARMONIC NUMBER (DEGREE) OF THE MULTIPOLE DATA BEING READ
C            IN. (NN=1 FOR DIPOLES, 2 FOR QUADRUPOLES, ETC.)

```

```
C     NM - THE NUMBER OF MULTIPOLES WITH HARMONIC NUMBER NN.


C     DO ** NJ=1,NM

C       P - THE POSITION VECTOR (IN INCHES) OF THE MULTIPOLE IN RECTANG-
C           ULAR COORDINATES.

C       A(I) - THE SPHERICAL COEFFICIENTS FOR THE MULTIPOLE OF DEGREE NN
C              WHERE I=1,2,--,NN+1. (I-1 IS THE ORDER OF THE ITH COEFF.)

C       B(I) - THE SPHERICAL COEFFICIENTS FOR THE MULTIPOLE OF DEGREE NN
C              WHERE I=1,2,--,NN. (I IS THE ORDER OF THE ITH COEFF.)

C       **NOTE** THE COEFFICIENTS FOR A DIPOLE OF MOMENT (DX,DY,DZ) ARE
C                 AS FOLLOWS --
C                               A(1)=DZ, A(2)=DX, AND B(1)=DY.

C   ** CONTINUE


C   ***********************************************************************
      DIMENSION F(1000),P(3),A(11),B(10),R(3),DM(3),F1(3),F9(7)
     +,A1(33),A2(1000)
      NAMELIST/NAM/NO,N1,NH,IP,R1,EG,EA,ED,PY
      DATA PI/3.14159265358979/
      IPT=0
   10 READ(5,01) NO,N1,NH,IP
      IF(IPT.EQ.0.AND.IP.NE.0) CALL CALCM1(0,10H024 LACKEY,-10.)
      IF(NO.EQ.0) GO TO 70
      IF(IP.NE.0) IPT=1
      READ(5,02) R1,EG,EA,ED,PY
      READ(5,04) F9
      WRITE(6,NAM)
      WRITE(6,14) F9
      D1=PI/FLOAT(NO)
      D2=2.*PI/FLOAT(N1-1)
      IZ=1
      DO 50 NI=1,NH
      READ(5,01) NN,NM
      WRITE(6,03) NN,NM
      NN1=NN+1
      DO 40 NJ=1,NM
      READ(5,02) P
      READ(5,F9) (A(I),I=1,NN1)
      READ(5,F9) (B(I),I=1,NN)
      WRITE(6,02) P
      WRITE(6,F9) (A(I),I=1,NN1)
      WRITE(6,F9) (B(I),I=1,NN)
      IF(NJ.EQ.1.AND.IZ.EQ.1) CALL SUM(0.,P,0.,P,DM)
      IF(NN.NE.1) GO TO 20
      DM(1)=DM(1)+A(2)
      DM(2)=DM(2)+B(1)
      DM(3)=DM(3)+A(1)
   20 DO 30 K=1,NO
      IF(IZ.EQ.1) A1(K)=FLOAT(K-1)*D1+(RANF(1.1)-.5)*2.*EA*PI/180.
      S1=SIN(A1(K))
      C1=COS(A1(K))
      DO 30 L=1,N1
      KL=L+(K-1)*N1
```

```
      IF(IZ.EQ.1) A2(KL)=FLOAT(L-1)*D2+(RANF(1.1)-.5)*2.*EA*PI/180.
      S2=SIN(A2(KL))
      C2=COS(A2(KL))
      IF(IZ.EQ.1) F(KL)=0.
      R(1)=R1*C1*S2
      R(2)=R1*S1*S2
      R(3)=R1*C2
      CALL AMPFLD(R,P,NN,A,B,F1)
   30 F(KL)=F(KL)+DOT(F1,R)/SQRT(DOT(R,R))
      IZ=0
   40 CONTINUE
   50 CONTINUE
      N3=NO*N1
      DO 60 I=1,N3
   60 F(I)=F(I)+ED*1.E-5+(RANF(1.1)-.5)*2.E-5*EG
      CALL PRNPUF(F,NO,N1)
      CALL SPCOOR(DM,F1)
      WRITE(6,06) DM,F1
      IF(IP.NE.0) CALL DATPLT(NO,N1,F,PY)
      GO TO 10
   70 IF(IPT.NE.0) CALL CALCM1(0,10H024 LACKEY,+10.)
      STOP
   01 FORMAT(14I5)
   02 FORMAT(9F8.4)
   03 FORMAT(48H1POSITION VECTOR AND COEFFICIENTS FOR NN ( NM = ,2I3,
     + 7H ARE --)
   04 FORMAT(7A10)
   06 FORMAT(20H1ACTUAL MOMENT IS --/3F20.12/3F20.12)
   14 FORMAT(6H0F9 = ,7A10)
      END

      SUBROUTINE AMPFLD(R,P,N,A,B,F)
      DIMENSION R(3),P(3),A(11),B(10),F(3),U(3),V(3),W(3)
      DATA PI/3.14159265358979/
      CALL SUM(1.,R,-1.,P,U)
      CALL SPCOOR(U,V)
      R1=V(1)*2.54
      IF(R1.NE.0.) GO TO 10
      PRINT 01
      RETURN
   10 O1=V(2)*PI/180.
      O2=V(3)*PI/180.
      S1=SIN(O1) $ S2=SIN(O2)
      C1=COS(O1) $ C2=COS(O2)
      U(1)=C1*S2 $ U(2)=S1*S2 $ U(3)=C2
      V(1)=C1*C2 $ V(2)=S1*C2 $ V(3)=-S2
      W(1)=-S1   $ W(2)=C1    $ W(3)=0.
      H1=H2=H3=0.
      NN=N+1
      DO 30 MM=1,NN
      M=MM-1
      P1=FLOAT(M)*O1
      SM1=SIN(P1) $ CM1=COS(P1)
      P1=SPNM(N,M,C2)
      P2=SPNM(N,M+1,C2)
      P3=P4=0. $ CP=SQRT(FLOAT(N*(N+1))/2.)
      IF(M.EQ.0) GO TO 20
      P4=FLOAT((N-M+1)*(N-M+2))*PNM(N+1,M-1,C2)+PNM(N+1,M+1,C2)
      P4=.5*SQRT(2.*ANF(N-M)/ANF(N+M))*P4
      P3=C2*P4 $ CP=SQRT(FLOAT((N-M)*(N+M+1)))
```

```
20 B1=0.
   IF(M.GT.0) B1=B(M)
   A1=A(MM)*CM1+B1*SM1
   H1=H1+A1*P1
   H2=H2+A1*(CP*P2-P3)
   H3=H3+(A(MM)*SM1-B1*CM1)*P4
30 CONTINUE
   R1=R1**(N+2)
   H1=FLOAT(N+1)*H1/R1 $ H2=H2/R1 $ H3=H3/R1
   CALL SUM(H1,U,H2,V,F) $ CALL SUM(1.,F,H3,W,F)
   RETURN
01 FORMAT(50H0FIELD VECTOR CANNOT BE COMPUTED AT POLE POSITION.//)
   END

   FUNCTION SPNM(N,M,X)
   SPNM=PNM(N,M,X)
   IF(M.EQ.0.OR.M.GT.N) RETURN
   SPNM=SQRT(2.*ANF(N-M)/ANF(N+M))*SPNM
   RETURN
   END

   FUNCTION PNM(N,M,X)
   PNM=0.
   IF(M.GT.N) RETURN
   IF((ABS(1.-ABS(X))).GT.1.E-9) GO TO 20
   IF(M.NE.0) RETURN
   PNM=-1.
   IF(X.GT.0..OR.N.EQ.INT(FLOAT(N)/2.+.1)*2) PNM=1.
   RETURN
20 CNM=(2.**N)*ANF(N)*ANF(N-M)
   PNM=1.
   IF(M.NE.0) PNM=SQRT(1.-X*X)**M
   CNM=ANF(2*N)*PNM/CNM
   PNM=1.
   IF(M.NE.N) PNM=X**(N-M)
   IF(N-M.LE.1) GO TO 40
   PRD1=1.
   NT=(N-M)/2
   DO 30 I=1,NT
   AN1=FLOAT(N-M-2*I+2)
   AN2=FLOAT(2*I)
   AN3=FLOAT(2*N-2*I+1)
   PRD1=-PRD1*AN1*(AN1-1)/(AN2*AN3)
   NE=N-M-2*I
   AN1=1.
   IF(NE.GT.0) AN1=X**NE
   PNM=PNM+PRD1*AN1
30 CONTINUE
40 PNM=CNM*PNM
   RETURN
   END

   FUNCTION ANF(N)
   DOUBLE PRECISION AN
   ANF=1.
   AN=1.D0
   IF(N.LT.0) PRINT 01
   IF(N.LT.2) RETURN
   DO 10 I=2,N
10 AN=AN*DBLE(FLOAT(I))
```

```
      ANF=SNGL(AN)
      RETURN
   01 FORMAT(37H1FACTORIAL INTEGER IS LESS THAN ZERO.//)
      END

      SUBROUTINE SPCOOR(D,R)
      DIMENSION D(3),R(3)
      DATA PI/3.14159265358979/
      R(1)=SQRT(DOT(D,D))
      R(2)=0.
      R(3)=0.
      IF(R(1).EQ.0.) RETURN
      IF(D(1)**2+D(2)**2.NE.0.) R(2)=ATAN2(D(2),D(1))*180./PI
      R(3)=ATAN2(SQRT(D(1)**2+D(2)**2),D(3))*180./PI
      RETURN
      END

      SUBROUTINE SUM(A,X,B,Y,Z)
      DIMENSION X(3),Y(3),Z(3)
      DO 10 I=1,3
   10 Z(I)=A*X(I)+B*Y(I)
      RETURN
      END

      FUNCTION DOT(X,Y)
      DIMENSION X(3),Y(3)
      DOT=X(1)*Y(1)+X(2)*Y(2)+X(3)*Y(3)
      RETURN
      END

      SUBROUTINE PRNPUF(F,NO,N1)
      DIMENSION F(800),F1(33),IF(800)
      N3=NO*N1
      ICS=10
      ICZ=0
      II=0
      DO 20 KO=1,NO
      KS=1+(KO-1)*N1
      KE=KS+N1-1
      DO 10 I=1,N1
      II=II+1
      F1(I)=F(II)*1.E5
   10 IF(II)=INT(F(II)*1.E+6)
      WRITE(6,01) KO
   20 WRITE(6,02) (F1(I),I=1,N1)
      WRITE(7,03) ICS,ICZ,(IF(I),I=1,N3)
   01 FORMAT(11H0CURVE NO. ,I2)
   02 FORMAT(1H ,8F8.1)
   03 FORMAT(9I6)
      RETURN
      END

      SUBROUTINE DATPLT(NO,N1,F,YDIST)
      DIMENSION F(1000),F1(33),F2(73),X(33),X1(73),C(1 ),P(1 ),CH(16)
      DATA PI/3.14159265358979/
      DATA CH/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13,
     + 2H14,2H15,2H16/
      N3=NO*N1
      CALL YSCALE(N3,F,YDIST,YMAX,YMIN)
      DX=360./FLOAT(N1-1)
```

```
      F1(1)=F1(2)=YMAX $ X(1)=0. $ X(2)=360.
      CALL CALCM1(2,X,F1,0,0.,360.,YMIN,YMAX,6.,7.,TITLE,0,
     + 20HCOLATITUDE (DEGREES),-20,30HNORMAL MAGNETIC FIELD (GAMMAS),
     + 30,0.,18)
      F1(2)=YMIN $ X(1)=X(2)=360.
      CALL CALCM1(-2,X,F1,0)
      F1(1)=F1(2)=0.    $ X(1)=360. $ X(2)=0.
      IF(YMAX.GT.0.0.AND.YMIN.LT.0.0) CALL CALCM1(-2,X,F1,0)
      DO 10 I=1,N1
   10 X(I)=DX*FLOAT(I-1)
      DO 20 I=1,73
   20 X1(I)=5.*FLOAT(I-1)
      IJ=0
      DO 50 I=1,NO
      DO 30 J=1,N1
      IJ=IJ+1
   30 F1(J)=F(IJ)*1.E+5
      CALL FNCTON(F1,N1-1,C,CO,P,N2,DEV)
      DO 40 N=1,N2
      EN=N
      DO 40 J=1,73
      XX=X1(J)*PI/180.
      IF(N.EQ.1) F2(J)=CO
   40 F2(J)=F2(J)+C(N)*SIN(EN*XX+P(N))
      CALL CALCM1(-73,X1,F2,0)
      DO 50 J=1,N1
      NC=1+I/10
      CALL SYMBL4(X(J)/60.,(F1(J)-YMIN)/YDIST,.08,CH(I),0.,NC)
   50 CONTINUE
      RETURN
      END

      SUBROUTINE FNCTON(F,NSCANS,C,CO,PHI,N2,DEV)
      DIMENSION F(1),G(33),C(1),PHI(1)
      DATA PI/3.14159265358979/
      DX=2.*PI/FLOAT(NSCANS)
      NE=NSCANS/2+1
      CO=0.
      DO 10 J=1,NSCANS
   10 CO=CO+F(J)/FLOAT(NSCANS)
      DO 30 N=1,NE
      EN=N
      S1=0.
      C1=0.
      X=-DX
      DO 20 J=1,NSCANS
      X=X+DX
      S1=S1+(1./PI)*F(J)*SIN(EN*X)*DX
   20 C1=C1+(1./PI)*F(J)*COS(EN*X)*DX
      C(N)=SQRT(S1**2+C1**2)
      IF((FLOAT(N)).EQ.(FLOAT(NSCANS)/2.)) C(N)=C(N)/2.
      PHI(N)=0.0
   30 IF((S1**2+C1**2).NE.0.) PHI(N)=ATAN2(C1,S1)
      DO 50 N=1,NE
      EN=N
      X=-DX
      DEV1=0.
      DO 40 J=1,NSCANS
      X=X+DX
      IF(N.EQ.1) G(J)=CO
```

```
      G(J)=G(J)+C(N)*SIN(EN*X+PHI(N))                                   SA403490
   40 IF(ABS(G(J)-F(J)).GT.DEV1) DEV1=ABS(G(J)-F(J))                    SA403500
      IF(N.EQ.1) DEV=DEV1                                               SA403510
      IF(N.EQ.1) N2=1                                                   SA403520
      IF(DEV1.LT.DEV) N2=N                                              SA403530
      IF(DEV1.LT.DEV) DEV=DEV1                                          SA403540
   50 CONTINUE                                                          SA403550
      RETURN                                                            SA403560
      END                                                               SA403570
                                                                        SA403580
      SUBROUTINE YSCALE(N,F,YD,YX,YM)                                   SA403590
      DIMENSION F(800)                                                  SA403600
      PP=F(1)                                                           SA403610
      PN=PP                                                             SA403620
      DO 10 I=1,N                                                       SA403630
      IF(F(I).GT.PP) PP=F(I)                                            SA403640
      IF(F(I).LT.PN) PN=F(I)                                            SA403650
   10 CONTINUE                                                          SA403660
      PP=PP*1.E+5                                                       SA403670
      PN=PN*1.E+5                                                       SA403680
      P=PP-PN                                                           SA403690
      SN=0.                                                             SA403700
      IF(PP.GT.0.) SN=1.                                                SA403710
      IF(YD.NE.0.) GO TO 50                                             SA403720
      DO 20 I=1,21                                                      SA403730
      FA=(1.E-9)*(10.**I)                                               SA403740
      FA1=FA*.1                                                         SA403750
      IF(P/FA.GE.1.) GO TO 20                                           SA403760
      GO TO 30                                                          SA403770
   20 CONTINUE                                                          SA403780
   30 YD=5.*FA1                                                         SA403790
      YX=FLOAT(INT(PP/YD+SN))*YD                                        SA403800
      YM=YX-7.*YD                                                       SA403810
      CN=4.                                                             SA403820
      DO 40 I=1,3                                                       SA403830
      CN=CN*.5                                                          SA403840
      YD1=CN*FA1                                                        SA403850
      YX1=FLOAT(INT(PP/YD1+SN))*YD1                                     SA403860
      YM1=YX1-7.*YD1                                                    SA403870
      IF((7.*YD1-P).LT.0.0.OR.YX1.LT.PP.OR.YM1.GT.PN) RETURN            SA403880
      YD=YD1                                                            SA403890
      YX=YX1                                                            SA403900
      YM=YM1                                                            SA403910
   40 CONTINUE                                                          SA403920
   50 YX=FLOAT(INT(PP/YD+SN))*YD                                        SA403930
      YM=YX-7.*YD                                                       SA403940
      RETURN                                                            SA403950
      END                                                               SA403960
```

APPENDIX H

SAMPLE PROBLEM FOR SA4024

I. PROBLEM EXECUTED ON INTERCOM (TIME-SHARING SYSTEM)

```
NOL INTERCOM
TYPE "LOGIN."
LOGIN.

TYPE USERNAME/PASSWORD/TTY NO. (OUTSIDE NOL TTY=88)
        024533LACK/[redacted]/4


 09/10/73    15.12.25. BD/42/35
C- SETUP.FORTRAN

ON AT       15.12.38. 09/10/73
**FORTRAN
**NEW OR OLD FILE- ATTACH(BN4024,BN4024)

15.12.56.ATTACH(BN4024,BN4024)
**READY.
REWIND(BN4024)*COPYBR(BN4024,FIL,9)*RETURN(BN4024)

**READY.
FIL.
     6    25     1     0
    96.00    1.000    0.200    25.00
    (1H ,7E10.4)







$NAM

NO        =   6,
```

```
N1        =  25,

NH        =  1,

IP        =  0,

R1        =  0.96E+02,

FC        =  0.1E+01,

FA        =  0.2E+00,

FD        =  0.25E+02,

FY        = -0.0,

$END

F9 =      (1H ,7E10.4)
      1     3




POSITION VECTOR AND COEFFICIENTS FOR NN & NM =    1   3 ARE --
    39.00    0.000    0.000
  +.0000E+00+.3100E+05
  +.0000E+00

39.0000   0.0000   0.0000
0.              .3100E+05
0.
   -21.00   36.373    0.000
  +.0000E+00-.1500E+05
  +.2598E+05


21.0000 36.3730   0.0000
0.           -.1500E+05
 .2598E+05
   -21.00 -36.373    0.000
  +.0000E+00-.1500E+05
  -.2598E+05
```

```
21.0000-36.3730  0.0000
0.          -.1500E+05
-.2598E+05

CURVE NO.  1
  -504.3  -475.6  -381.5  -123.3   438.5  1273.8  1749.2  1259.7
   431.3  -125.8  -382.2  -476.4  -502.8  -499.8  -476.5  -436.6
  -386.3  -341.0  -322.6  -339.4  -386.6  -436.0  -476.8  -499.1
  -503.8

CURVE NO.  2
  -502.8  -481.3  -414.6  -275.5   -20.8   281.0   427.2   280.3
   -26.8  -272.8  -415.6  -480.7  -504.2  -497.6  -451.0  -328.7
   -83.8   235.4   407.3   239.5   -80.6  -329.0  -450.5  -497.1
  -503.8

CURVE NO.  3
  -502.5  -487.8  -454.5  -406.6  -348.0  -296.8  -278.0  -297.1
  -348.4  -406.6  -455.4  -488.3  -503.1  -492.7  -419.0  -190.5
   405.3  1389.2  2019.5  1395.2   393.6  -193.1  -419.7  -491.0
  -504.3

CURVE NO.  4
  -503.5  -491.6  -442.4  -319.4   -81.9   238.1   399.1   233.5
   -77.1  -320.2  -442.6  -492.0  -504.1  -490.6  -440.9  -315.1
   -74.8   253.5   420.1   253.7   -74.5  -318.2  -441.7  -491.1
  -503.7

CURVE NO.  5
  -502.5  -492.5  -420.0  -188.4   397.7  1389.0  2018.3  1392.8
   390.7  -192.5  -419.2  -491.1  -503.6  -488.4  -456.9  -405.6
  -347.5  -298.5  -277.5  -298.0  -347.8  -406.3  -454.5  -488.6
  -503.4

CURVE NO.  6
  -502.4  -497.0  -452.0  -327.9   -86.4   229.7   401.3   237.1
   -82.9  -329.9  -448.4  -497.0  -503.7  -481.8  -415.6  -274.3
   -22.6   286.3   433.2   279.6   -20.7  -271.3  -414.7  -481.6
  -503.6




ACTUAL MØMENT IS --
  1000.000000000000     0.000000000000     0.000000000000
  1000.000000000000     0.000000000000    90.000000000000
```

```
    0    0    0    0
15.20.24.STØP
**READY.
LØGØUT.

CP TIME      4.233
PP TIME     56.828
CØNNECT TIME  0 HR  8 MIN 11 SEC
TØTAL CØST ØF SESSIØN = $   2.60
 09/10/73  LØGGED ØUT AT 15.20.36.<
```

Notes:

1. The file BN4024 is the binary version of SA4024. It consists of 12 binary records (subprograms). The last three are plotting routines and are not used when executing problems on INTERCOM.

2. The information typed in by the user has been underlined.

APPENDIX H (CONT.)

II. LISTING OF DAT024

```
   10      0 -5043 -4755 -3815 -1233  4384 12737 17492
12596   4313 -1257 -3821 -4763 -5027 -4998 -4765 -4365
-3863 -3410 -3226 -3394 -3865 -4359 -4767 -4990 -5037
-5028 -4812 -4146 -2754  -208  2810  4272  2802  -267
-2727 -4155 -4806 -5041 -4976 -4510 -3287  -838  2354
 4072  2395  -806 -3289 -4504 -4971 -5037 -5025 -4877
-4544 -4066 -3480 -2968 -2779 -2971 -3484 -4066 -4554
-4883 -5030 -4926 -4190 -1905  4053 13892 20194 13952
 3935 -1930 -4196 -4910 -5042 -5035 -4916 -4423 -3194
 -819  2381  3990  2334  -770 -3201 -4425 -4920 -5041
-4906 -4408 -3151  -748  2534  4201  2537  -745 -3181
-4416 -4911 -5037 -5024 -4924 -4199 -1884  3977 13889
20183 13927  3907 -1925 -4192 -4911 -5036 -4884 -4568
-4056 -3474 -2985 -2775 -2979 -3477 -4063 -4545 -4886
-5034 -5024 -4970 -4520 -3278  -863  2297  4012  2371
 -829 -3299 -4484 -4970 -5036 -4817 -4156 -2743  -225
 2863  4332  2796  -207 -2712 -4146 -4815 -5035
```

<u>Notes</u>:

1. The data is stored in DAT024 in a way that allows the file to be read in a (9F6.1) format.

2. The first two data points on the first line are values representing the calibration and zero readings, CS and CZ. The value of the calibration signal CV should be set to 1.0 when using data generated from SA4024.

APPENDIX H (CONT.)

III. PROBLEM SUBMITTED TO BATCH

```
NØL INTERCØM
TYPE "LØGIN."
LØGIN(S)
024533LACK/[redacted]/4


 09/11/73    09.43.58. BC/42/34
C- SETUP.GENERAL

ØN AT      09.44.23. 09/11/73
**GENERAL
**NEW ØR ØLD FILE- NEW/IECC024*TAPE(ØN)

**READY.


1IECC4ST,P1,T200,CM060000.55302435,024,LACKEY.
2ATTACH(ABC,NØLBIN)
3CØPYN(0,DEF,ABC)
4RETURN(ABC)
5ATTACH(BN4024,BN4024)
6REWIND(BN4024)
7CØPYBF(BN4024,CBA)
8RETURN(BN4024)
9LØAD(CBA)
10DEF.
11*WFØR
12REWIND(ABC)
13GØULD1,14,ABC
14*WFØR
6100     6    25      1      1
6110    96.00    1.000    0.200    25.00    0.000
6120    (1H ,7E10.4)
6130     1     3
6140    39.00    0.000    0.000
6150 +.0000E+00+.3100E+05
6160 +.0000E+00
6170  -21.00   36.373    0.000
6160 +.0000E+00-.1500E+05
```

Tape prepared beforehand in LOCAL mode

```
6190 +.2598E+05
6200  -21.00 -36.373   0.000
6210 +.0000E+00-.1500E+05
6220 -.2598E+05
6230    6   25     1     1
6240   96.00   0.000   0.000   0.000   0.000
6250   (1H ,7E10.4)
6260    1    1
6270   0.000   0.000   0.000
6280 +.0000E+00+.1000E+04
6290 +.0000E+00
6300    6   25     1     1
6310   96.00   0.000   0.000   0.000   0.000
6320   (1H ,7E10.4)
6330    2    1
6340   0.000   0.000   0.000
6350 -.9472E+07+.0000E+00-.2241E+06
6360 +.0000E+00+.0000E+00
6370    6   25     1     1
6380   96.00   0.000   0.000   0.000   0.000
6390   (1H ,7E10.4)
6400    3    1
6410   0.000   0.000   0.000
6420 +.0000E+00+.6837E+08+.0000E+00+.2341E+10
6430 +.0000E+00+.0000E+00+.0000E+00
6440    6   25     1     1
6450   96.00   0.000   0.000   0.000   0.000
6460   (1H ,7E10.4)
6470    4    1
6480   0.000   0.000   0.000
6490 +.1545E+12+.0000E+00+.1406E+11+.0000E+00-.1860E+11
6500 +.0000E+00+.0000E+00+.0000E+00+.0000E+00
6510    6   25     1     1
6520   96.00   0.000   0.000   0.000   0.000
6530   (1H ,7E10.4)
6540    5    1
6550   0.000   0.000   0.000
6560 +.0000E+00-.2180E+13+.0000E+00-.2812E+14+.0000E+00-.3158E+13
6570 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00
6580    6   25     1     1
6590   96.00   0.000   0.000   0.000   0.000
6600   (1H ,7E10.4)
6610    6    1
6620   0.000   0.000   0.000
6630 -.2109E+16+.0000E+00-.3228E+15+.0000E+00+.3536E+15+.0000E+00+.4533E
+16
6640 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00
6650    6   25     1     1
6660   96.00   0.000   0.000   0.000   0.000
6670   (1H ,7F10.4)
6680    7    1
6690   0.000   0.000   0.000
6700 +.0000E+00+.4320E+17+.0000E+00+.3541E+18+.0000E+00+.4962E+17+.0000E
+00
```

Tape prepared beforehand in LOCAL Mode

```
6710 -.6764E+17
6720 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E
+00
6730    6   25    1    1
6740   96.00   0.000   0.000   0.000   0.000
6750   (1H ,7E10.4)
6760    8    1
6770   0.000   0.000   0.000
6780 +.2699E+20+.0000E+00+.5698E+19+.0000E+00-.5975E+19+.0000E+00-.4517E
+20
6790 +.0000E+00-.9102E+19
6800 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000F+00+.0000E
+00
6810 +.0000E+00
6820    6   25    8    1
6830   96.00   0.000   0.000   0.000   0.000
6840   (1H ,7F10.4)
6850    1    1
6860   0.000   0.000   0.000
6870 +.0000E+00+.1000E+04
6880 +.0000E+00
6890    2    1
6900   0.000   0.000   0.000
6910 -.9472E+07+.0000E+00-.2241E+06
6920 +.0000E+00+.0000E+00
6930    3    1
6940   0.000   0.000   0.000
6950 +.0000E+00+.6837E+08+.0000E+00+.2341E+10
6960 +.0000E+00+.0000E+00+.0000E+00
6970    4    1
6980   0.000   0.000   0.000
6990 +.1545E+12+.0000E+00+.1406E+11+.0000E+00-.1860E+11
7000 +.0000E+00+.0000E+00+.0000E+00+.0000E+00
7010    5    1
7020   0.000   0.000   0.000
7030 +.0000E+00-.2180E+13+.0000E+00-.2812E+14+.0000E+00-.3158E+13
7040 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00
7050    6    1
7060   0.000   0.000   0.000
7070 -.2109E+16+.0000E+00-.3228E+15+.0000E+00+.3536E+15+.0000E+00+.4533E
+16
7080 +.0000E+00+.0000F+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00
7090    7    1
7100   0.000   0.000   0.000
7110 +.0000E+00+.4320E+17+.0000E+00+.3541E+18+.0000E+00+.4962E+17+.0000E
+00
7120 -.6764E+17
7130 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E
+00
7140    8    1
7150   0.000   0.000   0.000
7160 +.2699E+20+.0000E+00+.5698E+19+.0000E+00-.5975E+19+.0000E+00-.4517E
+20
```

Tape prepared beforehand in LOCAL mode

```
7170 +.0000E+00-.9102E+19
7180 +.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E+00+.0000E
+00
7190 +.0000E+00
7200     0     0     0     0
TAPE(ØFF)*SAVE*PURGE(IECC024)*BATCH.*QUEUES.
```

Tape prepared beforehand in LOCAL mode

```
**SAVED IECC024
09.56.18.PURGE(IECC024)
TYPE FILE NAME-IECC024

TYPE DISPØSITIØN-INPUT

TYPE FILE NAME-END


QUEUES  09.57.13. I= 18, Ø=   2, P=   0, C=   4.
INPUT = 18
IECC43L-1 CABRE3J-4 GFJLK3K-5 GRID73I-5 GRIDN3H-5 CABRE3A-4
CCBCB3F-5 CABRE3E-4 CABRE3G-4 CABRE3D-4 EAADW3B-5 HHJC027-3
CABRE25-4 IECC426-1 CABAP3C-5 HHJ1J22-2 CABRE28-4 CESEQ02-3
ØUTPUT=  2
AJFLZ1Z-0 HHJDP19-0
PUNCH = NØNE
CØMMØN=  4
GEØPP  -0 KEEP7  -0 SSSSSSU-0 SSSSSST-0
CØNTRØL PTS.
DAJRI24-5 DBA1820-1
09.57.13.STØP
**READY.
LØGØUT.

CP TIME      1.674
PP TIME     82.029
CØNNECT TIME  0 HR 14 MIN  9 SEC
TØTAL CØST ØF SESSIØN = $    3.58
 09/11/73  LØGGED ØUT AT 09.58.07.<
```

Note:

1. The information typed in by the user has been underlined.

APPENDIX I

LISTING OF SA5024

```
      PROGRAM DIPSTD(INPUT=65,OUTPUT=65,TAPE5=INPUT,TAPE6=OUTPUT,TAPE99)


C                          SPHERICAL HARMONIC PROGRAM


C       THIS PROGRAM GENERATES AND ANALYZES DATA REPRESENTING THE NORMAL
C     COMPONENT OF THE MAGNETIC FIELD FROM A SATELLITE. THE DATA IS ENTERED
C     IN THE FOLLOWING ORDER --

C     NO - THE NUMBER OF GREAT CIRCLES OF DATA. (NO IS USUALLY EVEN, E.G.,
C          NO=(N1-1)/2. THE PROGRAM STOPS IF NO=0.)

C     N1 - THE NUMBER OF DATA POINTS PER GREAT CIRCLE. (N1 IS ALWAYS ODD.
C          THE FIRST DATA POINT IS THE SAME AS THE LAST FOR EACH GREAT CIR-
C          LE.)

C     NH1 - THE TOTAL NUMBER OF DIFFERENT HARMONICS (DEGREES) OF MULTIPOLE
C           MAGNETS TO BE CONSIDERED.

C     NH2 - THE HARMONIC NUMBER (DEGREE) REPRESENTING THE HIGHEST DEGREE
C           SPHERICAL HARMONIC TERM TO BE COMPUTED FROM THE DATA. (NH2=1
C           FOR DIPOLES, 2 FOR QUADRUPOLES, ETC.)

C     IP1 - DETERMINES WHETHER OR NOT THE MAGNETIC DATA IS TO BE PLOTTED.
C          (IP1=0 MEANS THAT THE DATA WILL NOT BE PLOTTED.)

C     IP2 - DETERMINES WHETHER OR NOT THE MAGNETIC DATA IS TO BE PRINTED.
C          (IP2=0 MEANS THAT THE DATA WILL NOT BE PRINTED.)

C     IW - DETERMINES THE TYPE OF INTEGRATING SCHEME TO BE USED. (IW=0
C          MEANS THAT THE EXACT, ALGEBRAIC SCHEME IS TO BE USED.)

C     R1 - THE RADIUS OF THE MEASUREMENT SPHERE IN INCHES.

C     EG - THE ERROR (IN GAMMAS) TO BE RANDOMLY INSERTED INTO THE DATA TO
C          REPRESENT INSTRUMENTATION INACCURACIES.

C     EA - THE ERROR (IN DEGREES) TO BE RANDOMLY INSERTED INTO THE DATA TO
C          REPRESENT MEASUREMENT POSITION ERRORS.

C     ED - THE CONSTANT ERROR (IN GAMMAS) TO BE INSERTED INTO THE DATA.
C          (THIS WILL BE ANALYZED AS MONOPOLE MOMENT.)

C     PY - THE SCALE FACTOR (GAMMAS/INCH) FOR THE Y-AXIS IF THE DATA IS TO
C          BE PLOTTED. (IF PY=0.0 A FACTOR WILL BE COMPUTED FROM THE DATA.)

C     F9 - THE FORMAT FOR READING AND PRINTING THE SPHERICAL COEFFICIENTS
```

```
C        A(I) AND B(I).


C  DO ** NI=1,NH1

C    NN - THE HARMONIC NUMBER (DEGREE) OF THE MULTIPOLE DATA BEING READ
C         IN. (NN=1 FOR DIPOLES, 2 FOR QUADRUPOLES, ETC.)

C    NM - THE NUMBER OF MULTIPOLES WITH HARMONIC NUMBER NN.


C    DO ** NJ=1,NM

C      P - THE POSITION VECTOR (IN INCHES) OF THE MULTIPOLE IN RECTANG-
C          ULAR COORDINATES.

C      A(I) - THE SPHERICAL COEFFICIENTS FOR THE MULTIPOLE OF DEGREE NN
C             WHERE I=1,2,--,NN+1. (I-1 IS THE ORDER OF THE ITH COEFF.)

C      B(I) - THE SPHERICAL COEFFICIENTS FOR THE MULTIPOLE OF DEGREE NN
C             WHERE I=1,2,--,NN (I IS THE ORDER OF THE ITH COEFF.)

C      **NOTE** THE COEFFICIENTS FOR A DIPOLE OF MOMENT (DX,DY,DZ) ARE
C               AS FOLLOWS --
C                            A(1)=DZ, A(2)=DX, AND B(1)=DY.

C  ** CONTINUE


C  ***********************************************************************
      DIMENSION F(1000),P(3),PV(200),A(11),B(10),R(3),DM(3),F1(3),D(3)
     +,A1(33),A2(1000),F9(7)
      NAMELIST/NAM/NO,N1,NH1,NH2,IP1,IP2,IW,R1,EG,EA,ED,PY
      DATA PI/3.14159265358979/
      IPT=0
   10 READ(5,01) NO,N1,NH1,NH2,IP1,IP2,IW
      IF(IPT.EQ.0.AND.IP1.NE.0) CALL CALCM1(0,10H024 LACKEY,-10.)
      IF(NO.EQ.0) GO TO 110
      IF(IP1.NE.0) IPT=1
      READ(5,02) R1,EG,EA,ED,PY
      READ(5,04) F9
      WRITE(6,NAM)
      WRITE(6,14) F9
      D1=PI/FLOAT(NO)
      D2=2.*PI/FLOAT(N1-1)
      IZ=1
      DO 50 NI=1,NH1
      READ(5,01) NN,NM
      WRITE(6,03) NN,NM
      NN1=NN+1
      DO 40 NJ=1,NM
      READ(5,02) P
      READ(5,F9) (A(I),I=1,NN1)
      READ(5,F9) (B(I),I=1,NN)
      WRITE(6,02) P
      WRITE(6,F9) (A(I),I=1,NN1)
      WRITE(6,F9) (B(I),I=1,NN)
      IF(NJ.EQ.1.AND.IZ.EQ.1) CALL SUM(0.,P,0.,P,DM)
      IF(NN.NE.1) GO TO 20
      DM(1)=DM(1)+A(2)
```

```
      DM(2)=DM(2)+B(1)
      DM(3)=DM(3)+A(1)
   20 DO 30 K=1,NO
      IF(IZ.EQ.1) A1(K)=FLOAT(K-1)*D1+(RANF(1.1)-.5)*2.*EA*PI/180.
      S1=SIN(A1(K))
      C1=COS(A1(K))
      DO 30 L=1,N1
      KL=L+(K-1)*N1
      IF(IZ.EQ.1) A2(KL)=FLOAT(L-1)*D2+(RANF(1.1)-.5)*2.*EA*PI/180.
      S2=SIN(A2(KL))
      C2=COS(A2(KL))
      IF(IZ.EQ.1) F(KL)=0.
      R(1)=R1*C1*S2
      R(2)=R1*S1*S2
      R(3)=R1*C2
      CALL AMPFLD(R,P,NN,A,B,F1)
   30 F(KL)=F(KL)+DOT(F1,R)/SQRT(DOT(R,R))
      IZ=0
   40 CONTINUE
   50 CONTINUE
      N3=NO*N1
      DO 60 I=1,N3
   60 F(I)=F(I)+ED*1.E-5+(RANF(1.1)-.5)*2.E-5*EG
      IF(IW.EQ.0) CALL WGT1(NO,N1,PV)
      IF(IW.NE.0) CALL WGT3(NO,N1,PV)
      CALL AMPMNT(NO,N1,0,R1,F,PV,A,B)
      WRITE(6,05) A(1)
      IF(IP2.NE.0) CALL PRINTF(F,NO,N1)
      IF(IP1.NE.0) CALL DATPLT(NO,N1,F,PY)
      DO 100 NI=1,NH2
      CALL AMPMNT(NO,N1,NI,R1,F,PV,A,B)
      IF(NI.GT.1) GO TO 80
      D(1)=A(2)
      D(2)=B(1)
      D(3)=A(1)
      CALL SPCOOR(D,R)
      CALL SPCOOR(DM,F1)
      WRITE(6,06) DM,F1
      WRITE(6,07) D,R
      IF(F1(1).EQ.0.) WRITE(6,08)
      IF(F1(1).EQ.0.) GO TO 90
      DO 70 I=1,3
   70 R(I)=(DM(I)-D(I))*100./F1(1)
      WRITE(6,09) R
      GO TO 90
   80 IF(NI.GT.2) GO TO 90
      S3=SQRT(3.)
      Q11=S3*A(3)-A(1)
      Q22=-S3*A(3)-A(1)
      Q33=-Q11-Q22
      Q12=S3*B(2)
      Q13=S3*A(2)
      Q23=S3*B(1)
      WRITE(6,11) Q11,Q22,Q33,Q12,Q13,Q23
   90 WRITE(6,12) NI
      NI1=NI+1
      WRITE(6,13) (A(I),I=1,NI1)
      WRITE(6,13) (B(I),I=1,NI)
  100 CONTINUE
      GO TO 10
```

```
110 IF(IPT.NE.0) CALL CALCM1(0,10H024 LACKEY,+10.)
    STOP
 01 FORMAT(14I5)
 02 FORMAT(9F8.4)
 03 FORMAT(48H1POSITION VECTOR AND COEFFICIENTS FOR NN ( NM = ,2I3,
   + 7H ARE --)
 04 FORMAT(7A10)
 05 FORMAT(26H0THE MONOPOLE MOMENT IS --,F20.6//)
 06 FORMAT(20H1ACTUAL MOMENT IS --/3F20.6/3F20.6)
 07 FORMAT(24H1CALCULATED MOMENT IS --/3F20.6/3F20.6)
 08 FORMAT(42H0PERCENT ERROR CANNOT BE COMPUTED - DM = 0)
 09 FORMAT(20H0PERCENT ERROR IS --/3E20.12)
 11 FORMAT(54H0THE QUADRUPOLE MOMENTS Q11,Q22,Q33,Q12,Q13,Q23 ARE --/
   + (3E20.12))
 12 FORMAT(27H0THE A ( B COEFFS. FOR THE ,I2,16HTH HARMONIC ARE9)
 13 FORMAT(1H ,5E20.12)
 14 FORMAT(6H0F9 = ,7A10)
    END

    SUBROUTINE AMPMNT(N0,N1,N,R1,F,P,A,B)
    DIMENSION F(1000),P(200),PN(10,200),A(11),B(10)
    DATA PI/3.14159265358979323846/
    CALL POLVAL(N1,N,PN)
    KL=0
    N2=(N1+1)/2
    D1=PI/FLOAT(N0)
    D2=2.*PI/FLOAT(N1-1)
    DO 10 K=1,N0
    A1=D1*FLOAT(K-1)
    LP=+1
    LI=0
    DO 10 L=1,N1
    KL=KL+1
    IF(L.GT.N2) LP=-1
    LI=LI+LP
    A2=A1+FLOAT(1-LP)*PI/2.
    F1=F(KL)
    A2N=FLOAT(2*N+1)
    A1N=FLOAT(N+1)
    AR=(R1*2.54)**(N+2)
    NN=N+1
    DO 10 MM=1,NN
    M=MM-1
    AM=FLOAT(M)
    IF(KL.EQ.1) A(MM)=0.
    IF(KL.EQ.1.AND.M.GT.0) B(M)=0.
    PNM1=PN(MM,LI)*COS(AM*A2)
    PNM2=PN(MM,LI)*SIN(AM*A2)
    AC=(A2N*AR/A1N)*F1*P(LI)
    A(MM)=A(MM)+AC*PNM1
    IF(M.GT.0) B(M)=B(M)+AC*PNM2
 10 CONTINUE
    RETURN
    END

    SUBROUTINE WGT1(N0,N1,P)
    DIMENSION D(50),P(200)
    N4=(N1+1)/4
    AN=FLOAT(N0)
    CALL WGT2(N1,D)
```

```
      DO 10 J1=1,N4
      J3=N1/2+2-J1
      DJ=D(J1)/(4.*AN)
      P(J1)=DJ
   10 P(J3)=DJ
      IF(INT(FLOAT(N1-1)/4.+.1)*4.NE.(N1-1)) GO TO 20
      J1=N4+1
      P(J1)=D(J1)/(4.*AN)
   20 N4=(N1+1)/2
      P(N4)=P(N4)*2.
      RETURN
      END

      SUBROUTINE WGT2(N,D)
      DIMENSION A(50,50),D(50),C(50)
      DOUBLE PRECISION A,C,PI,AN
      DATA PI/3.14159265358979323846264 3D0/
      N3=(N+1)/4
      N4=N3+1
      DO 10 I=2,N4
      C(I)=1./FLOAT(2*I-1)
      A(I,1)=1.
      A(I,N4)=0.
      DO 10 J=2,N3
      AN=2.*PI*DBLE(FLOAT(J-1))/DBLE(FLOAT(N-1))
   10 A(I,J)=(DCOS(AN))**(2*I-2)
      C(1)=2.
      A(1,N4)=1.
      DO 20 J=1,N3
   20 A(1,J)=2.
      N3=N4
      IF(INT(FLOAT(N-1)/4.+.1)*4.NE.(N-1)) N3=N3-1
      CALL GAUSEL(A,C,N3,N3,1)
      DO 30 I=1,N3
   30 D(I)=C(I)
      RETURN
      END

      SUBROUTINE WGT3(N0,N1,P)
      DIMENSION P(200)
      DATA PI/3.14159265358979323846/
      D2=2.*PI/FLOAT(N1-1)
      E1=(SIN(D2/4.))**2
      E2=SIN(D2/2.)
      P(1)=E1/(2.*FLOAT(N0))
      N2=(N1+1)/2
      P(N2)=P(1)*2.
      IF(N1.LE.3) RETURN
      N3=N2-1
      DO 10 L=2,N3
      S2=SIN(D2*FLOAT(L-1))
      P(L)=ABS(S2*E2)/(2.*FLOAT(N0))
   10 CONTINUE
      RETURN
      END

      SUBROUTINE GAUSEL(A,C,M,N,IT)
      DIMENSION A(50,50),C(50),IB(50)
      DOUBLE PRECISION A,C,D
C     WRITE(6,03)
```

```
C     DO 20 I=1,M                                                 SA502890
C  20 WRITE(6,04) (A(I,J),J=1,N),C(I)                             SA502900
      DO 70 I=1,N                                                 SA502910
   70 IB(I)=I                                                     SA502920
      DO 60 I=1,M                                                 SA502930
      IF(IT.NE.0.AND.M.NE.N) GO TO 35                             SA502940
      D=0.                                                        SA502950
      JJ=0                                                        SA502960
      DO 10 J=I,N                                                 SA502970
      IF(DABS(A(I,J)).LE.D) GO TO 10                              SA502980
      JJ=J                                                        SA502990
      D=DABS(A(I,J))                                              SA503000
   10 CONTINUE                                                    SA503010
      IF(JJ.EQ.I) GO TO 35                                        SA503020
      IF(JJ.NE.0) GO TO 140                                       SA503030
      WRITE(6,01) I                                               SA503040
      GO TO 110                                                   SA503050
  140 DO 160 J=1,M                                                SA503060
      D=A(J,JJ)                                                   SA503070
      A(J,JJ)=A(J,I)                                              SA503080
  160 A(J,I)=D                                                    SA503090
      ID=IB(JJ)                                                   SA503100
      IB(JJ)=IB(I)                                                SA503110
      IB(I)=ID                                                    SA503120
   35 D=A(I,I)                                                    SA503130
      DO 30 J=I,N                                                 SA503140
   30 A(I,J)=A(I,J)/D                                             SA503150
      C(I)=C(I)/D                                                 SA503160
      DO 50 J=1,M                                                 SA503170
      IF(J.EQ.I) GO TO 50                                         SA503180
      D=A(J,I)                                                    SA503190
      IF(D.EQ.0.) GO TO 50                                        SA503200
      DO 40 K=I,N                                                 SA503210
   40 A(J,K)=A(J,K)-D*A(I,K)                                      SA503220
      C(J)=C(J)-D*C(I)                                            SA503230
   50 CONTINUE                                                    SA503240
   60 CONTINUE                                                    SA503250
  110 IF(IT.EQ.0.OR.M.NE.N) GO TO 100                             SA503260
      DO 120 I=1,N                                                SA503270
  120 A(I,N)=C(I)                                                 SA503280
      DO 130 I=1,N                                                SA503290
      II=IB(I)                                                    SA503300
  130 C(II)=A(I,N)                                                SA503310
  100 CONTINUE                                                    SA503320
C     IF(IT.EQ.0) WRITE(6,02) (IB(I),I=1,N)                       SA503330
C     WRITE(6,05)                                                 SA503340
C     DO 80 I=1,M                                                 SA503350
C  80 WRITE(6,04) (A(I,J),J=1,N),C(I)                             SA503360
      RETURN                                                      SA503370
   01 FORMAT(5H1ROW ,I2,14H IS ALL ZEROS.)                        SA503380
   02 FORMAT(24H0SINGLE PERMUTATION IS...,20I3//)                 SA503390
   03 FORMAT(21H0THE INPUT MATRIX IS9//)                          SA503400
   04 FORMAT(1H ,7D16.8)                                          SA503410
   05 FORMAT(22H0THE OUTPUT MATRIX IS9//)                         SA503420
      END                                                         SA503430
                                                                  SA503440
      SUBROUTINE POLVAL(N1,N,PN)                                  SA503450
      DIMENSION PN(10,200)                                        SA503460
      DATA PI/3.14159265358979323846/                             SA503470
      D2=2.*PI/FLOAT(N1-1)                                        SA503480
```

```
      N2=(N1+1)/2
      DO 10 I=1,N2
      A2=D2*FLOAT(I-1)
      C2=COS(A2)
      NN=N+1
      DO 10 MM=1,NN
      M=MM-1
      PN(MM,I)=SPNM(N,M,C2)
   10 CONTINUE
      RETURN
      END

      SUBROUTINE AMPFLD(R,P,N,A,B,F)
      DIMENSION R(3),P(3),A(11),B(10),F(3),U(3),V(3),W(3)
      DATA PI/3.14159265358979/
      CALL SUM(1.,R,-1.,P,U)
      CALL SPCOOR(U,V)
      R1=V(1)*2.54
      IF(R1.NE.0.) GO TO 10
      PRINT 01
      RETURN
   10 O1=V(2)*PI/180.
      O2=V(3)*PI/180.
      S1=SIN(O1) $ S2=SIN(O2)
      C1=COS(O1) $ C2=COS(O2)
      U(1)=C1*S2 $ U(2)=S1*S2 $ U(3)=C2
      V(1)=C1*C2 $ V(2)=S1*C2 $ V(3)=-S2
      W(1)=-S1    $ W(2)=C1    $ W(3)=0.
      H1=H2=H3=0.
      NN=N+1
      DO 30 MM=1,NN
      M=MM-1
      P1=FLOAT(M)*O1
      SM1=SIN(P1) $ CM1=COS(P1)
      P1=SPNM(N,M,C2)
      P2=SPNM(N,M+1,C2)
      P3=P4=0. $ CP=SQRT(FLOAT(N*(N+1))/2.)
      IF(M.EQ.0) GO TO 20
      P4=FLOAT((N-M+1)*(N-M+2))*PNM(N+1,M-1,C2)+PNM(N+1,M+1,C2)
      P4=.5*SQRT(2.*ANF(N-M)/ANF(N+M))*P4
      P3=C2*P4 $ CP=SQRT(FLOAT((N-M)*(N+M+1)))
   20 B1=0.
      IF(M.GT.0) B1=B(M)
      A1=A(MM)*CM1+B1*SM1
      H1=H1+A1*P1
      H2=H2+A1*(CP*P2-P3)
      H3=H3+(A(MM)*SM1-B1*CM1)*P4
   30 CONTINUE
      R1=R1**(N+2)
      H1=FLOAT(N+1)*H1/R1 $ H2=H2/R1 $ H3=H3/R1
      CALL SUM(H1,U,H2,V,F) $ CALL SUM(1.,F,H3,W,F)
      RETURN
   01 FORMAT(50H0FIELD VECTOR CANNOT BE COMPUTED AT POLE POSITION.//)
      END

      FUNCTION SPNM(N,M,X)
      SPNM=PNM(N,M,X)
      IF(M.EQ.0.OR.M.GT.N) RETURN
      SPNM=SQRT(2.*ANF(N-M)/ANF(N+M))*SPNM
      RETURN
```

```
      END

      FUNCTION PNM(N,M,X)
      PNM=0.
      IF(M.GT.N) RETURN
      IF((ABS(1.-ABS(X))).GT.1.E-9) GO TO 20
      IF(M.NE.0) RETURN
      PNM=-1.
      IF(X.GT.0..OR.N.EQ.INT(FLOAT(N)/2.+.1)*2) PNM=1.
      RETURN
   20 CNM=(2.**N)*ANF(N)*ANF(N-M)
      PNM=1.
      IF(M.NE.0) PNM=SQRT(1.-X*X)**M
      CNM=ANF(2*N)*PNM/CNM
      PNM=1.
      IF(M.NE.N) PNM=X**(N-M)
      IF(N-M.LE.1) GO TO 40
      PRD1=1.
      NT=(N-M)/2
      DO 30 I=1,NT
      AN1=FLOAT(N-M-2*I+2)
      AN2=FLOAT(2*I)
      AN3=FLOAT(2*N-2*I+1)
      PRD1=-PRD1*AN1*(AN1-1)/(AN2*AN3)
      NE=N-M-2*I
      AN1=1.
      IF(NE.GT.0) AN1=X**NE
      PNM=PNM+PRD1*AN1
   30 CONTINUE
   40 PNM=CNM*PNM
      RETURN
      END

      FUNCTION ANF(N)
      DOUBLE PRECISION AN
      ANF=1.
      AN=1.D0
      IF(N.LT.0) PRINT 01
      IF(N.LT.2) RETURN
      DO 10 I=2,N
   10 AN=AN*DBLE(FLOAT(I))
      ANF=SNGL(AN)
      RETURN
   01 FORMAT(37H1FACTORIAL INTEGER IS LESS THAN ZERO.//)
      END

      SUBROUTINE SPCOOR(D,R)
      DIMENSION D(3),R(3)
      DATA PI/3.14159265358979/
      R(1)=SQRT(DOT(D,D))
      R(2)=0.
      R(3)=0.
      IF(R(1).EQ.0.) RETURN
      IF(D(1)**2+D(2)**2.NE.0.) R(2)=ATAN2(D(2),D(1))*180./PI
      R(3)=ATAN2(SQRT(D(1)**2+D(2)**2),D(3))*180./PI
      RETURN
      END

      SUBROUTINE SUM(A,X,B,Y,Z)
      DIMENSION X(3),Y(3),Z(3)
```

```
      DO 10 I=1,3
   10 Z(I)=A*X(I)+B*Y(I)
      RETURN
      END

      FUNCTION DOT(X,Y)
      DIMENSION X(3),Y(3)
      DOT=X(1)*Y(1)+X(2)*Y(2)+X(3)*Y(3)
      RETURN
      END

      SUBROUTINE PRINTF(F,NO,N1)
      DIMENSION F(800),F1(100)
      DO 20 KO=1,NO
      DO 10 I=1,N1
      II=I+(KO-1)*N1
   10 F1(I)=F(II)*1.E5
      WRITE(6,01) KO
   20 WRITE(6,02) (F1(KK),KK=1,N1)
   01 FORMAT(11HOCURVE NO. ,I2)
   02 FORMAT(1H ,8F8.1)
      RETURN
      END

      SUBROUTINE DATPLT(NO,N1,F,YDIST)
      DIMENSION F(1000),F1(33),F2(73),X(33),X1(73),C(1 ),P(1 ),CH(16)
      DATA PI/3.14159265358979/
      DATA CH/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13,
     + 2H14,2H15,2H16/
      N3=NO*N1
      CALL YSCALE(N3,F,YDIST,YMAX,YMIN)
      DX=360./FLOAT(N1-1)
      F1(1)=F1(2)=YMAX $ X(1)=0. $ X(2)=360.
      CALL CALCM1(2,X,F1,0,0.,360.,YMIN,YMAX,6.,7.,TITLE,0,
     + 20HCOLATITUDE (DEGREES),-20,30HNORMAL MAGNETIC FIELD (GAMMAS),
     + 30,0.,18)
      F1(2)=YMIN $ X(1)=X(2)=360.
      CALL CALCM1(-2,X,F1,0)
      F1(1)=F1(2)=0.    $ X(1)=360. $ X(2)=0.
      IF(YMAX.GT.0.0.AND.YMIN.LT.0.0) CALL CALCM1(-2,X,F1,0)
      DO 10 I=1,N1
   10 X(I)=DX*FLOAT(I-1)
      DO 20 I=1,73
   20 X1(I)=5.*FLOAT(I-1)
      IJ=0
      DO 50 I=1,NO
      DO 30 J=1,N1
      IJ=IJ+1
   30 F1(J)=F(IJ)*1.E+5
      CALL FNCTON(F1,N1-1,C,CO,P,N2,DEV)
      DO 40 N=1,N2
      EN=N
      DO 40 J=1,73
      XX=X1(J)*PI/180.
      IF(N.EQ.1) F2(J)=CO
   40 F2(J)=F2(J)+C(N)*SIN(EN*XX+P(N))
      CALL CALCM1(-73,X1,F2,0)
      DO 50 J=1,N1
      NC=1+I/10
      CALL SYMBL4(X(J)/60.,(F1(J)-YMIN)/YDIST,.08,CH(I),0.,NC)
```

```
50 CONTINUE
   RETURN
   END

   SUBROUTINE FNCTON(F,NSCANS,C,CO,PHI,N2,DEV)
   DIMENSION F(1),G(33),C(1),PHI(1)
   DATA PI/3.14159265358979/
   DX=2.*PI/FLOAT(NSCANS)
   NE=NSCANS/2+1
   CO=0.
   DO 10 J=1,NSCANS
10 CO=CO+F(J)/FLOAT(NSCANS)
   DO 30 N=1,NE
   EN=N
   S1=0.
   C1=0.
   X=-DX
   DO 20 J=1,NSCANS
   X=X+DX
   S1=S1+(1./PI)*F(J)*SIN(EN*X)*DX
20 C1=C1+(1./PI)*F(J)*COS(EN*X)*DX
   C(N)=SQRT(S1**2+C1**2)
   IF((FLOAT(N)).EQ.(FLOAT(NSCANS)/2.)) C(N)=C(N)/2.
   PHI(N)=0.0
30 IF((S1**2+C1**2).NE.0.) PHI(N)=ATAN2(C1,S1)
   DO 50 N=1,NE
   EN=N
   X=-DX
   DEV1=0.
   DO 40 J=1,NSCANS
   X=X+DX
   IF(N.EQ.1) G(J)=CO
   G(J)=G(J)+C(N)*SIN(EN*X+PHI(N))
40 IF(ABS(G(J)-F(J)).GT.DEV1) DEV1=ABS(G(J)-F(J))
   IF(N.EQ.1) DEV=DEV1
   IF(N.EQ.1) N2=1
   IF(DEV1.LT.DEV) N2=N
   IF(DEV1.LT.DEV) DEV=DEV1
50 CONTINUE
   RETURN
   END

   SUBROUTINE YSCALE(N,F,YD,YX,YM)
   DIMENSION F(800)
   PP=F(1)
   PN=PP
   DO 10 I=1,N
   IF(F(I).GT.PP) PP=F(I)
   IF(F(I).LT.PN) PN=F(I)
10 CONTINUE
   PP=PP*1.E+5
   PN=PN*1.E+5
   P=PP-PN
   SN=0.
   IF(PP.GT.0.) SN=1.
   IF(YD.NE.0.) GO TO 50
   DO 20 I=1,21
   FA=(1.E-9)*(10.**I)
   FA1=FA*.1
   IF(P/FA.GE.1.) GO TO 20
```

```
      GO TO 30                                                          SA505890
   20 CONTINUE                                                          SA505900
   30 YD=5.*FA1                                                         SA505910
      YX=FLOAT(INT(PP/YD+SN))*YD                                        SA505920
      YM=YX-7.*YD                                                       SA505930
      CN=4.                                                             SA505940
      DO 40 I=1,3                                                       SA505950
      CN=CN*.5                                                          SA505960
      YD1=CN*FA1                                                        SA505970
      YX1=FLOAT(INT(PP/YD1+SN))*YD1                                     SA505980
      YM1=YX1-7.*YD1                                                    SA505990
      IF((7.*YD1-P).LT.0.0.OR.YX1.LT.PP.OR.YM1.GT.PN) RETURN            SA506000
      YD=YD1                                                            SA506010
      YX=YX1                                                            SA506020
      YM=YM1                                                            SA506030
   40 CONTINUE                                                          SA506040
   50 YX=FLOAT(INT(PP/YD+SN))*YD                                        SA506050
      YM=YX-7.*YD                                                       SA506060
      RETURN                                                            SA506070
      END                                                               SA506080
```

APPENDIX J

SAMPLE PROBLEMS FOR SA5024

I. PROBLEM EXECUTED ON INTERCOM (TIME-SHARING SYSTEMS)

```
NØL INTERCØM
TYPE "LØGIN."
LØGIN(S)
024533LACK/[redacted]/4


 09/14/73   13.26.48. BC/42/35
C- SETUP.GENERAL

ØN AT     13.27.14. 09/14/73
**GENERAL
**NEW ØR ØLD FILE- ATTACH(AAA,BN5024)*REWIND(AAA)*CØPYBR(AAA,FIL,15)

13.27.44.ATTACH(AAA,BN5024)
**READY.
RETURN(AAA)

**READY.
FIL.
     16    33     1     8     0     0     0
     96.00   0.000   0.000   0.000
     (7E10.4)




$NAM

NO       =   16,

N1       =   33,

NH1      =   1,
```

```
NH2      =  8,

IP1      =  0,

IP2      =  0,

IW       =  0,

R1       =  0.96E+02,

EG       =  0.0,

EA       =  0.0,

ED       =  0.0,

PY       = -0.0,

$END

F9 =     (7E10.4)
     1      3
```

```
POSITION VECTOR AND COEFFICIENTS FOR NN & NM =    1   3 ARE --
    39.00    0.000    0.000
+.0000E+00+.3100E+05
+.0000E+00

39.0000  0.0000  0.0000

.           .3100E+05

.
  -21.00   36.373   0.000
+.0000E+00-.1500E+05
+.2598E+05


21.0000 36.3730  0.0000

.          -.1500E+05
.2598E+05
```

```
  -21.00 -36.373   0.000
+.0000E+00-.1500E+05
-.2598E+05


21.0000-36.3730  0.0000

        -.1500E+05


.2598E+05

THE MØNØPØLE MØMENT IS --                .000005
```

```
ACTUAL MØMENT IS --
        1000.000000          0.000000          0.000000
        1000.000000          0.000000         90.000000
```

```
CALCULATED MØMENT IS --
         999.999347           .000000          -.000000
         999.999347           .000000         90.000000

PERCENT ERRØR IS --
  .653140392387E-04  -.367553809610E-09   .204352090805E-10

THE A & B CØEFFS. FØR THE  1TH HARMØNIC ARE:
  -.204352090805E-09   .999999346860E+03
   .367553809610E-08

THE QUADRUPØLE MØMENTS Q11,Q22,Q33,Q12,Q13,Q23 ARE --
  .908333976792E+07   .985968240459E+07  -.189430221725E+08
 -.140861894026E-05   .277251221753E-08  -.114908029088E-06
```

```
THE A & B CØEFFS. FØR THE  2TH HARMØNIC ARE:
  -.947151108626E+07   .160071067512E-08  -.224110815131E+06
  -.663421815261E-07  -.813266524347E-06

THE A & B CØEFFS. FØR THE  3TH HARMØNIC ARE:
   .468939542770E-04   .683659431170E+08  -.223517417908E-07   .23409232
3833E+10
  -.814609229565E-04   .884430482984E-05  -.183412397746E-03

THE A & B CØEFFS. FØR THE  4TH HARMØNIC ARE:
   .154465924862E+12  -.776198242188E-03   .140584755795E+11  -.10529816
1507E-01  -.186018026985E+11
   .351142883301E-02   .118371725082E+00  -.733375549316E-03  -.24567153
3048E-01

THE A & B CØEFFS. FØR THE  5TH HARMØNIC ARE:
  -.487304687500E+00  -.217954001634E+13  -.256347656250E-01  -.28122218
7225E+14  -.820884704590E-01
  -.315797097447E+13
   .177050781250E+01   .128906250000E+00   .177172851563E+01  -.77190017
7002E+00  -.294870560169E+02

THE A & B CØEFFS. FØR THE  6TH HARMØNIC ARE:
  -.210884991589E+16  -.195000000000E+02  -.322787259184E+15   .10126562
  -.210884991589E+16  -.195000000000E+02  -.322787259184E+15   .10126562
5000E+03   .353625883682E+15
   .225479736328E+02   .453272232211E+16
   .128687500000E+03   .104384375000E+04   .885156250000E+01   .17665273
4375E+04  -.262702636719E+02
  -.114442594910E+04

THE A & B CØEFFS. FØR THE  7TH HARMØNIC ARE:
  -.225424000000E+06   .431994422628E+17   .172800000000E+04   .35406399
7182E+18  -.101790000000E+05
   .496241920808E+17  -.293837539062E+05  -.676359128706E+17
  -.267840000000E+05  -.154800000000E+05  -.230640000000E+05  -.12074000
0000E+05   .115108750000E+06
  -.933500781250E+04  -.660728401367E+06

THE A & B CØEFFS. FØR THE  8TH HARMØNIC ARE:
   .269942772159E+20   .206438400000E+07   .569785815443E+19  -.60129280
0000E+07  -.597539013375E+19
  -.156819200000E+07  -.451694469107E+20   .765785875000E+06  -.91015764
9666E+19
  -.425574400000E+07   .626974720000E+08   .271462400000E+07   .52427776
0000E+08   .175494400000E+07
   .531896320000E+08  -.174570300000E+07  -.105260966187E+09
0

13.38.54.STØP
**READY.
LØGØUT.
```

```
CP TIME     22.322
PP TIME    106.909
CØNNECT TIME  0 HR 12 MIN 22 SEC
TØTAL CØST ØF SESSIØN = $    6.16
 09/14/73  LØGGED ØUT AT 13.39.10.<
```

Notes:

1. The file BN5024 is the binary version of SA5024. It consists of 18 binary records (subprograms). The last three are plotting routines and are not used when executing problems on INTERCOM.

2. The information typed in by the user has been underlined.

APPENDIX J (CONT.)

II. PROBLEM SUBMITTED TO BATCH

```
NØL INTERCØM
TYPE "LØGIN."
LØGIN(S)
024533LACK/[redacted]/4

 09/14/73    13.40.17. BC/42/35
C- SETUP.GENERAL

ØN AT      13.40.33. 09/14/73
**GENERAL
**NEW ØR ØLD FILE- NEW/IECC024*TAPE(ØN)

**READY.
1IECC5ST,P1,T200,CM060000.55302435,024,LACKEY.
2ATTACH(ABC,NØLBIN)
3CØPYN(O,DEF,ABC)
4RETURN(ABC)
5ATTACH(BN5024,BN5024)
6REWIND(BN5024)
7CØPYBF(BN5024,CBA)
8RETURN(BN5024)
9LØAD(CBA)
10DEF.
11*WEØR
12REWIND(ABC)
13GØULD1,14,ABC
14*WEØR
6100     16     33      1      8      1      1      0
6110     96.00    0.000    0.000    0.000    0.000
6120 (9F8.4)
6130      1      1
6140     20.00    0.000    0.000
6150     0.000    1000.
6160     0.000
6170     16     33      1      8      1      1      0
6180     96.00    0.000    0.000    0.000    0.000
6190 (9F8.4)
6200      1      1
6210     20.00    0.000    0.000
6220     0.000    0.000
6230     1000.
6240      0      0      0      0      0      0      0
TAPE(ØFF)
```

Tape prepared beforehand in LOCAL mode

```
**READY.
SAVE*PURGE(IECC024)*BATCH.*QUEUES.

**SAVED IECC024
13.43.50.PURGE(IECC024)
TYPE FILE NAME-IECC024

TYPE DISPØSITIØN-INPUT

TYPE FILE NAME-END


QUEUES  13.44.38. I=  7, Ø=  0, P=  0, C=  3.
INPUT =  7
CABAP72-2 BDCRØ7K-1 BDC9R7N-3 BDCSH7U-1 IECC579-1 DCCRW78-5
CESEQ02-3
ØUTPUT= NØNE
PUNCH = NØNE
CØMMØN=  3
FARE2  -0 SSSSSSU-0 SSSSSST-0
CØNTRØL PTS.
ICCBB63-3 GRIDA7S-5 CABTR7T-5 CBCQ26K-1 AUDIT73-4 HHJL371-2
CCB7870-5
13.44.38.STØP
**READY.
LØGØUT.

CP TIME       .785
PP TIME     96.846
CØNNECT TIME  0 HR  5 MIN  4 SEC
TØTAL CØST ØF SESSIØN = $   2.14
 09/14/73  LØGGED ØUT AT 13.45.21.<
```

Note:

1. **The information typed in by the user has been underlined.**

APPENDIX K

NOL TECHNICAL NOTE 9726
"NUMERICAL COMPUTATION OF LEGENDRE POLYNOMIALS AND SPHERICAL FUNCTIONS"

TN 9726

FOR INTERNAL USE ONLY

NAVAL ORDNANCE LABORATORY
White Oak, Silver Spring, Maryland

533:MHL:bag

NUMERICAL COMPUTATION OF LEGENDRE POLYNOMIALS AND SPHERICAL FUNCTIONS

7 November 1972

M. H. Lackey
Magnetic Structures Group

Asgmt: Task No. NOL-786/NPL (Magnetic Calibration for NPL)
TWP No. 530-524

Abst: This report describes computer subroutines for the generation of Legendre polynomials and spherical functions. The subroutines are coded in the FORTRAN IV computer language. A brief description of the functions and some of their properties is also included. A comparison is given of the numerical stability of two different methods of generating the functions.

Ref: (a) S. Chapman & J. Bartels, Geomagnetism (Oxford Press, London, 1940), Vol. II
(b) W. D. Macmillan, The Theory of the Potential (McGraw-Hill Book Co., New York, 1930)
(c) ASM 55 Handbook of Mathematical Functions (U.S. Government Printing Office, Washington, D.C., 1964)
(d) NOLTR 69-60, The Reaction of a Rigid Body to a Uniform Force Field, 17 Mar 1969, M. H. Lackey

Encl: (1) Figure 1, Regular Legendre Polynomials $P_n(\cos\theta)$ For n = 0 through 7
(2) Figure 2, Associated Legendre Polynomials $P_{7,m}(\cos\theta)$ For n = 0 through 7
(3) Figure 3, Schmidt Polynomials $P_7^m(\cos\theta)$ For m = 0 through 7
(4) Figure 4, Unstable Procedure for $P_n^{n-1}(\cos\theta)$ For n = 15 through 20
(5) Figure 5, Stable Procedure for $P_n^{n-1}(\cos\theta)$ For n = 15 through 20
(6) Appendix A, Listing of Function Subroutines

INTRODUCTION

1. Computer subroutines have been devised to compute values for three special types of polynomials including regular and associated Legendre polynomials, and Schmidt functions. The functions have special orthogonality properties which make them especially useful for interpolation and approximation. Techniques in spherical harmonic analysis can be defined in terms of the associated Legendre polynomials (spherical functions). The computer subroutines can be used to develop numerical methods for the harmonic analysis techniques. A listing of the subroutines in FORTRAN IV is included in Appendix A.

2. The subroutines are based on a finite series definition for the polynomials. It was determined that the series definition produced a more stable method than methods based on recurrence formulas.

POLYNOMIAL DEFINITIONS

3. There are a variety of ways to define the polynomials depending on the use intended for them. The definitions and terminology used in the following definitions conform to the usage in references (a) and (b). Some of the recurrence formulas were obtained from reference (c).

4. The regular Legendre polynomials $P_n(x)$ can be defined on the interval $-1 \le x \le 1$ by Rodrique's formula,

$$P_n(x) = \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n (x^2-1)^n \tag{1}$$

for $n \ge 0$ and $|x| \le 1$. The associated Legendre polynomials or spherical functions $P_{n,m}(x)$ can now be defined as

$$P_{n,m}(x) = (1-x^2)^{m/2}\left(\frac{d}{dx}\right)^m P_n(x) = \frac{(1-x^2)^{m/2}}{2^n n!}\left(\frac{d}{dx}\right)^{m+n}(x^2-1)^n \tag{2}$$

for $m \ge 0$, $n \ge 0$, and $|x| \le 1$. The index n designates the degree of the polynomial and the index m designates the order. It should be noted that some definitions of $P_{n,m}(x)$ contain a factor of $(-1)^m$ depending on whether the polynomials are considered to be functions of x on the interval $-1 \le x \le 1$ or as functions of θ (with $x = \cos\theta$) on the interval $0 \ge \theta \le \pi$. In this report the factor is dropped in keeping with the definitions in reference (a).

5. A comparison of Equations (1) and (2) show that the regular Legendre polynomials $P_n(x)$ are a subset of the associated Legendre polynomials $P_{n,m}(x)$, i.e., $P_n(x) = P_{n,0}(x)$. Also Equation (2) leads to the fact that $P_{n,m}(x) = 0$ for $m > n$.

6. Graphs of typical families of the polynomials are shown in Figures 1 and 2.

The regular Legendre polynomials shown in Figure 1 include P_0, P_1, ..., P_7. The polynomial values $P_n(\cos\theta)$ are plotted as functions of θ for $0 \le \theta \le 180^\circ$. Figure 2 shows the family $P_{7,m}(\cos\theta)$ for $m = 0, 1, 2, \ldots, 7$. Notice the variation in the value of the peaks of each curve. The curves for $m < 4$ appear as straight lines coinciding with the zero axis. This extensive variation in the order of magnitude for different polynomials $P_{n,m}(x)$ has disadvantages, especially in the development of numerical analysis procedures. This led Schmidt (see reference (a)) to develop a new set of polynomials.

7. The <u>Schmidt polynomials $P_n^m(x)$</u> are defined by scaling the associated Legendre polynomials as follows

$$\left\{ \begin{array}{l} P_n^0(x) = P_{n,0}(x) = P_n(x) \\ P_n^m(x) = \left\{ \dfrac{2\cdot(n-m)!}{(n+m)!} \right\}^{1/2} P_{n,m}(x) \end{array} \right\} \tag{3}$$

for $m > 0$ and $|x| \le 1$. The resulting polynomials all have values of the same order of magnitude. Also, $|P_n^m(x)| \le 1$ for $|x| \le 1$. A typical family of Schmidt polynomials $P_7^m(\cos\theta)$ for $m = 0, 1, 2, \ldots, 7$ is shown in Figure 3.

NUMERICAL METHODS

8. There are many ways of numerically generating sets of polynomials. One of the most common methods is the use of recurrence formulas. The associated Legendre polynomials satisfy several recurrence formulas including

$$P_{n,m}(x) = [(2n-1)\cdot x\cdot P_{n-1,m}(x) - (n+m-1)P_{n-2,m}(x)]/(n-m) \text{ for } m\neq n \text{ and } n>1 \tag{4}$$

$$P_{n,m}(x) = [(m-n-1)\cdot x\cdot P_{n,m-1}(x) + (n+m-1)P_{n-1,m-1}(x)]/(1-x^2)^{1/2} \text{ for } |x|<1, \; m>0, \text{ and } n>0 \tag{5}$$

$$P_{n,m}(x) = 2(m-1)\cdot x\cdot(1-x^2)^{-1/2}P_{n,m-1} - (n+m-1)(n-m+2)P_{n,m-2}(x) \text{ for } |x|<1 \text{ and } m>1. \tag{6}$$

Equation (4) represents a recurrence formula with varying degree, Equation (5) represents both varying degree and varying order, and Equation (6) represents varying order.

9. Another method for generating the associated Legendre polynomials is given

in reference (a). The polynomials are expressed as a finite (alternating) series in powers of x. The expression is:

$$P_{n,m}(x) = \frac{(2n)!(1-x^2)^{m/2}}{2^n \cdot n!(n-m)!} \left\{ x^{n-m} - \frac{(n-m)(n-m-1)x^{n-m-2}}{2(2n-1)} \right. \tag{7}$$

$$\left. + \frac{(n-m)(n-m-1)(n-m-2)(n-m-3)x^{n-m-4}}{2 \cdot 4(2n-1)(2n-3)} - \dots \right\}$$

where the last term inside the bracket is constant if (n-m) is even; or is a multiple of x if (n-m) is odd.

NUMERICAL STABILITY

10. Several different methods based on the recurrence formulas were used to test the numerical stability for large values of n and m, and for small values of (x^2-1). The only recurrence formula with any stability was Equation (4) which has varying degree. This formula can be used by itself only to generate the regular Legendre polynomials. When either Equation (5) or (6) is added the method becomes unstable. For example, Figure 4 shows a family of $P_n^{n-1}(\cos\theta)$ for n = 15, 16, ..., 20. Notice that instability becomes worse as $|x|$ approaches 1 and as n increases.

11. Instead of trying to stabilize the methods using the recurrence formulas it was decided to try the series Equation (7). Although the equation appears to be unstable itself (alternating series can be unstable) the results indicate the opposite. Figure 5 shows the same polynomials displayed in Figure 4 but generated by Equation (7). There seems to be no problem for $|x|$ near one or for large n. It was therefore decided to use Equation (7) for the subroutine. The computing time using Equation (7) was about the same as methods using recurrence formulas.

SUBROUTINE CALL STATEMENTS

12. The polynomial values can be computed as follows: Assume that y is a variable used in a computer program and that y is to be set equal to a polynomial value. Then

$y = P_n(x)$ is written as Y = PNM(N,0,X) for N≥0 and $|x| \le 1$.

$y = P_{n,m}(x)$ is written as Y = PNM(N,M,X) for N≥0, M≥0, and $|x| \le 1$.

$y = P_n^m(x)$ is written as Y = SPNM(N,M,X) for N≥0, M≥0, and $|x| \le 1$.

There are three function subprograms in the package. These include:

FUNCTION SPNM(N,M,X)

FUNCTION PNM(N,M,X)

FUNCTION ANF(N)

The subroutine ANF(N) is the factorial subroutine, i.e,

$y = n!$ is written as Y = ANF(N) for N≥0.

The factorial value is return as a floating point number based on the integer N. The subroutine SPNM uses both PNM and ANF as external functions. The subroutine PNM uses ANF as an external function. The subroutine ANF requires no external functions (machine functions only).

USES OF THE FUNCTIONS $P_n^m(x)$

13. It can be shown that the set of polynomials $\{P_n^m(x)\}$ of fixed order m are orthogonal on the interval $-1 \le x \le 1$ for all $n \ge m$. In fact:

$$\int_{-1}^{+1} P_n^m(x) P_{n'}^m(x)dx = \delta_{nn'} \frac{1}{2n+1} \tag{8}$$

for $n \ge m$ where $\delta_{nn'}$ is the Kronecker delta. The polynomials can therefore be normalized and used for interpolation and expansion of arbitrary functions on the interval $-1 \le x \le 1$.

14. The polynomials are of primary importance in areas of spherical harmonic analysis and potential theory. If we define two new sets of functions as

$$\{\sin(m\varphi)P_n^m(\cos\theta)\},\ \{\cos(m\varphi)P_n^m(\cos\theta)\} \tag{9}$$

and consider φ and θ to be the spherical angles representing longitude and colatitude respectively, then the functions are called spherical surface harmonics, and are orthogonal and complete on the surface of the unit sphere. They satisfy the condition

$$\int_0^{\pi}\int_0^{2\pi} \left[P_n^m(\cos\theta)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}\right] \left[P_{n'}^{m'}(\cos\theta)\begin{Bmatrix}\cos m'\varphi\\ \sin m'\varphi\end{Bmatrix}\right]\sin\theta d\theta d\varphi = \delta_{nn'}\delta_{mm'}\left\{\frac{4\pi}{2n+1}\right\} \tag{10}$$

for n, n', m, m' $\geq$ 0 (see reference (a)). Then any continuous function $f(\theta,\varphi)$ on the surface of the unit sphere can be expanded into a uniformly convergent series of surface harmonics as

$$f(\theta,\varphi) = \sum_{n=0}^{\infty} S_n(\theta,\varphi) = \sum_{n=0}^{\infty} \left\{ \sum_{m=0}^{n} (A_n^m \cos m\varphi + B_n^m \sin m\varphi) P_n^m(\cos\theta) \right\} \qquad (11)$$

where $S_n(\theta,\varphi)$ represents a general surface harmonic of degree n. The constants A_n^m and B_n^m are determined from the equations

$$\begin{Bmatrix} A_n^m \\ B_n^m \end{Bmatrix} = \frac{2n+1}{4\pi} \int_0^{\pi} \int_0^{2\pi} f(\theta,\varphi) P_n^m(\cos\theta) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix} \sin\theta \, d\varphi \, d\theta \qquad (12)$$

for m $\geq$ 0 and n $\geq$ 0.

15. Numerical integrating schemes for uniformly spaced data $f_{ij} = f(\theta_i,\varphi_j)$ on the surface of the unit sphere are discussed in reference (d).

SUMMARY

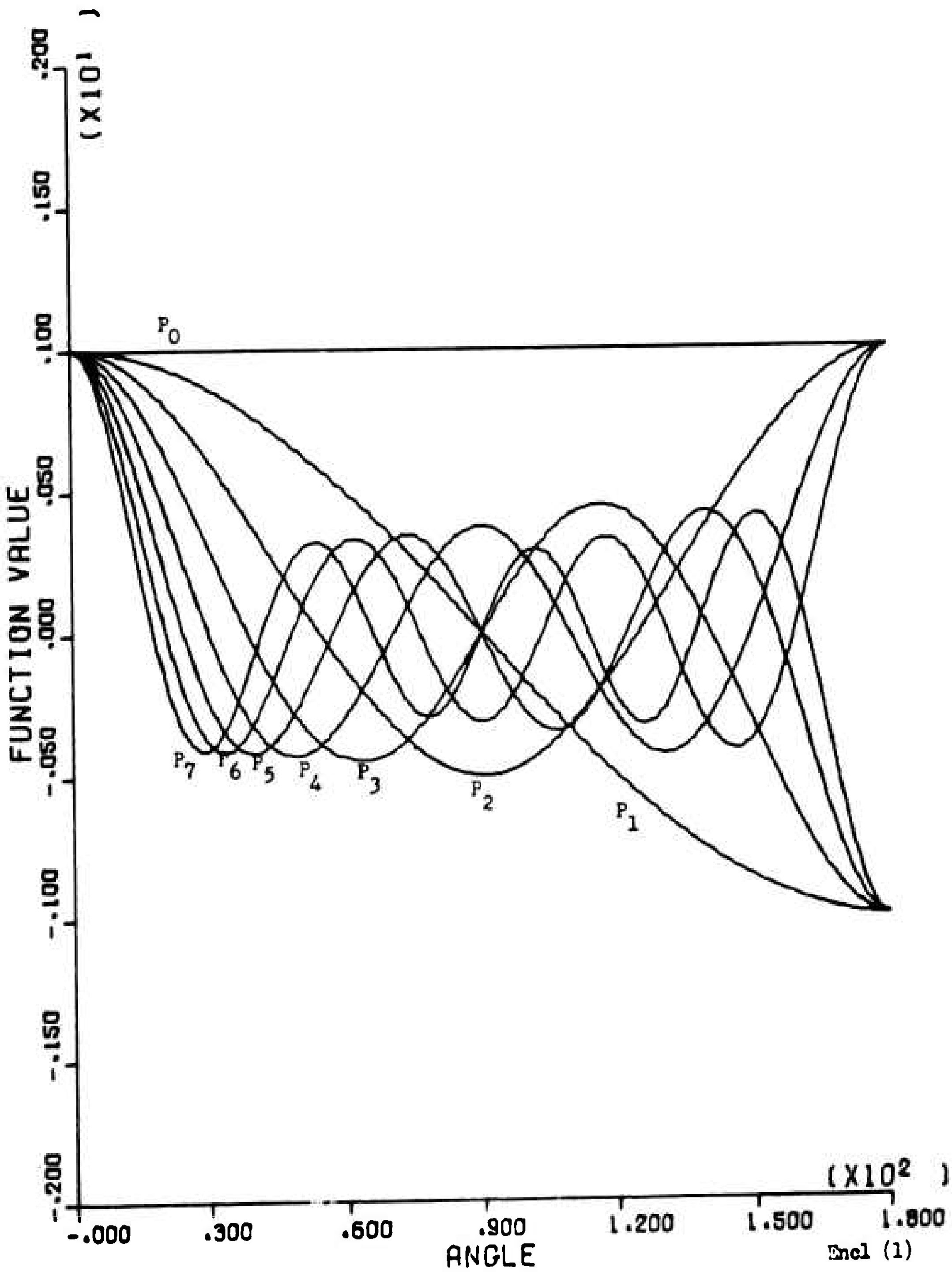
16. This report has discussed briefly some of the properties of Legendre polynomials and some of their uses. The subroutines listed in Appendix A can be used in a variety of ways including interpolation, approximation, and spherical harmonic analysis. Plans are being made to develop other subroutines to perform the spherical harmonic analysis and extrapolation of potential functions.

M. H. Lackey

M. H. LACKEY

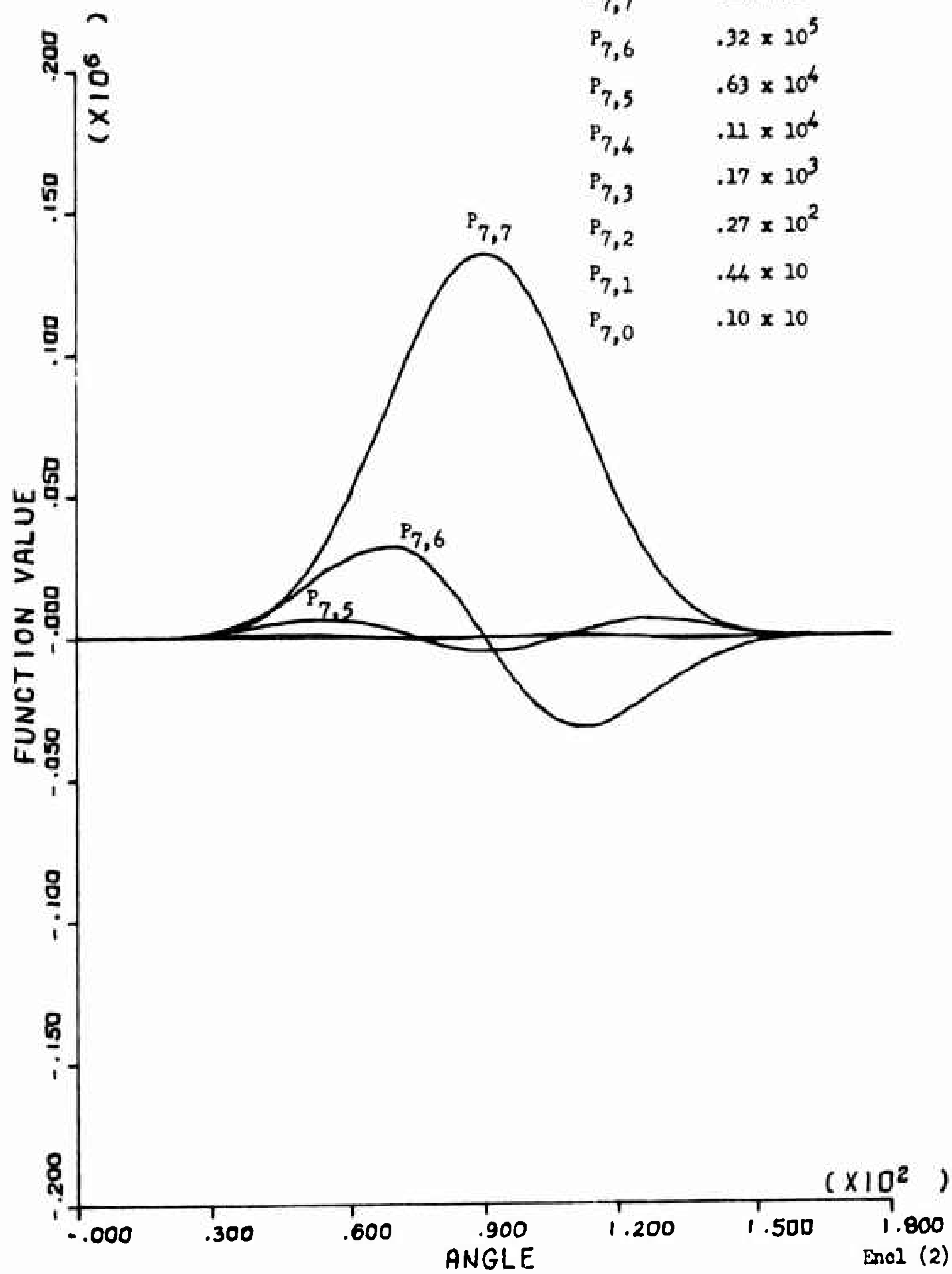
Copy to:
331 (R. E. Ferguson)
331 (T. A. Orlow)
530
531 (W. H. Wertman)
5332
720
720 (N. M. Ginsberg)

FIG. 1. REGULAR LEGENDRE POLYNOMIALS $P_n(\cos\upsilon)$ FOR n = 0 THROUGH 7



Encl (1)

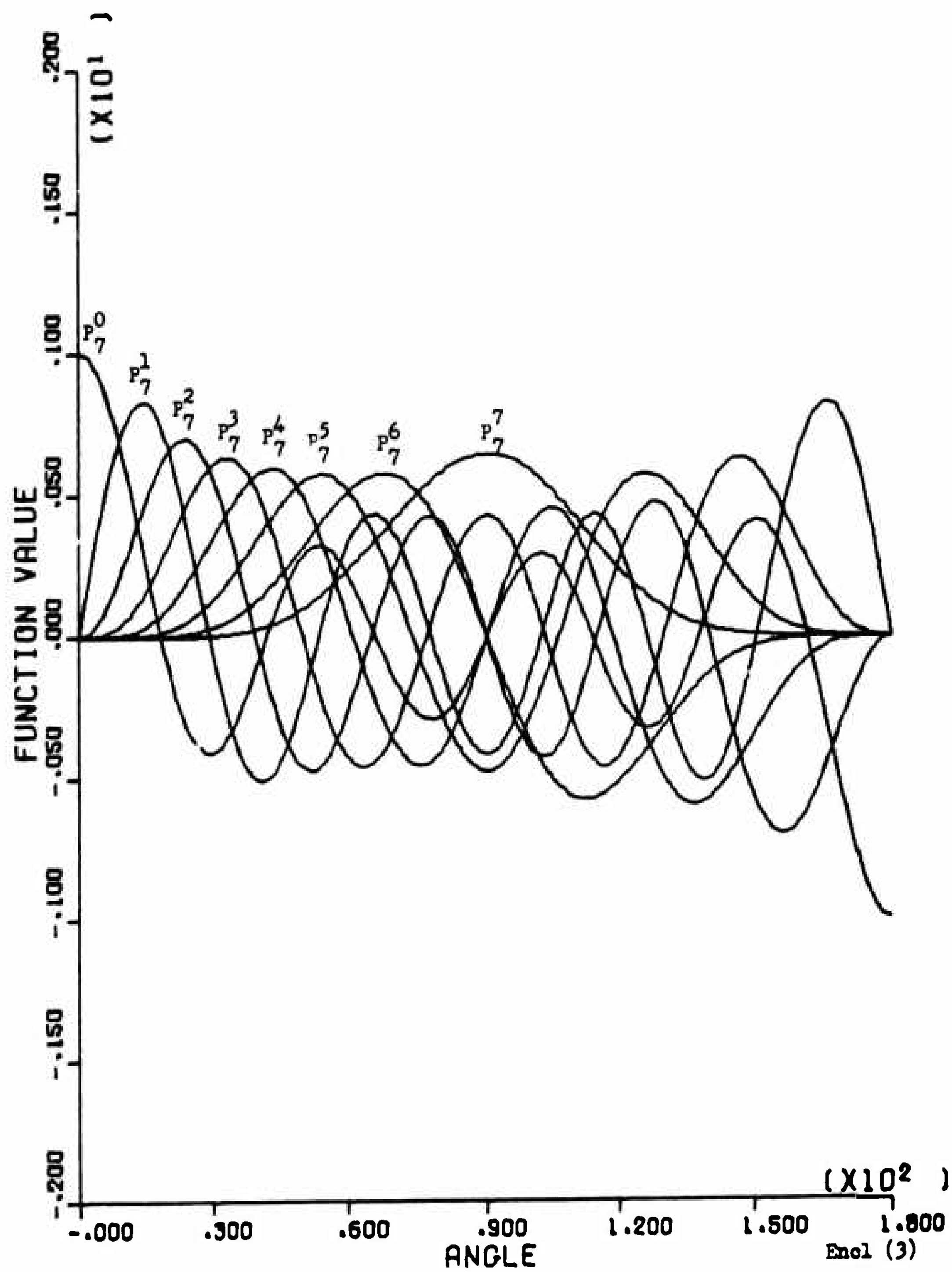
FIG. 2. ASSOCIATED LEGENDRE POLYNOMIALS $P_{7,m}(\cos\theta)$ FOR m = 0 THROUGH 7



Function	Approx. Peak
$P_{7,7}$	$.14 \times 10^6$
$P_{7,6}$	$.32 \times 10^5$
$P_{7,5}$	$.63 \times 10^4$
$P_{7,4}$	$.11 \times 10^4$
$P_{7,3}$	$.17 \times 10^3$
$P_{7,2}$	$.27 \times 10^2$
$P_{7,1}$	$.44 \times 10$
$P_{7,0}$	$.10 \times 10$

Encl (2)

FIG. 3. SCHMIDT POLYNOMIALS $P_7^m(\cos\theta)$ FOR m = 0 THROUGH 7



Encl (3)

FIG. 4. UNSTABLE PROCEDURE FOR $P_n^{n-1}(\cos\theta)$ FOR n = 15 THROUGH 20

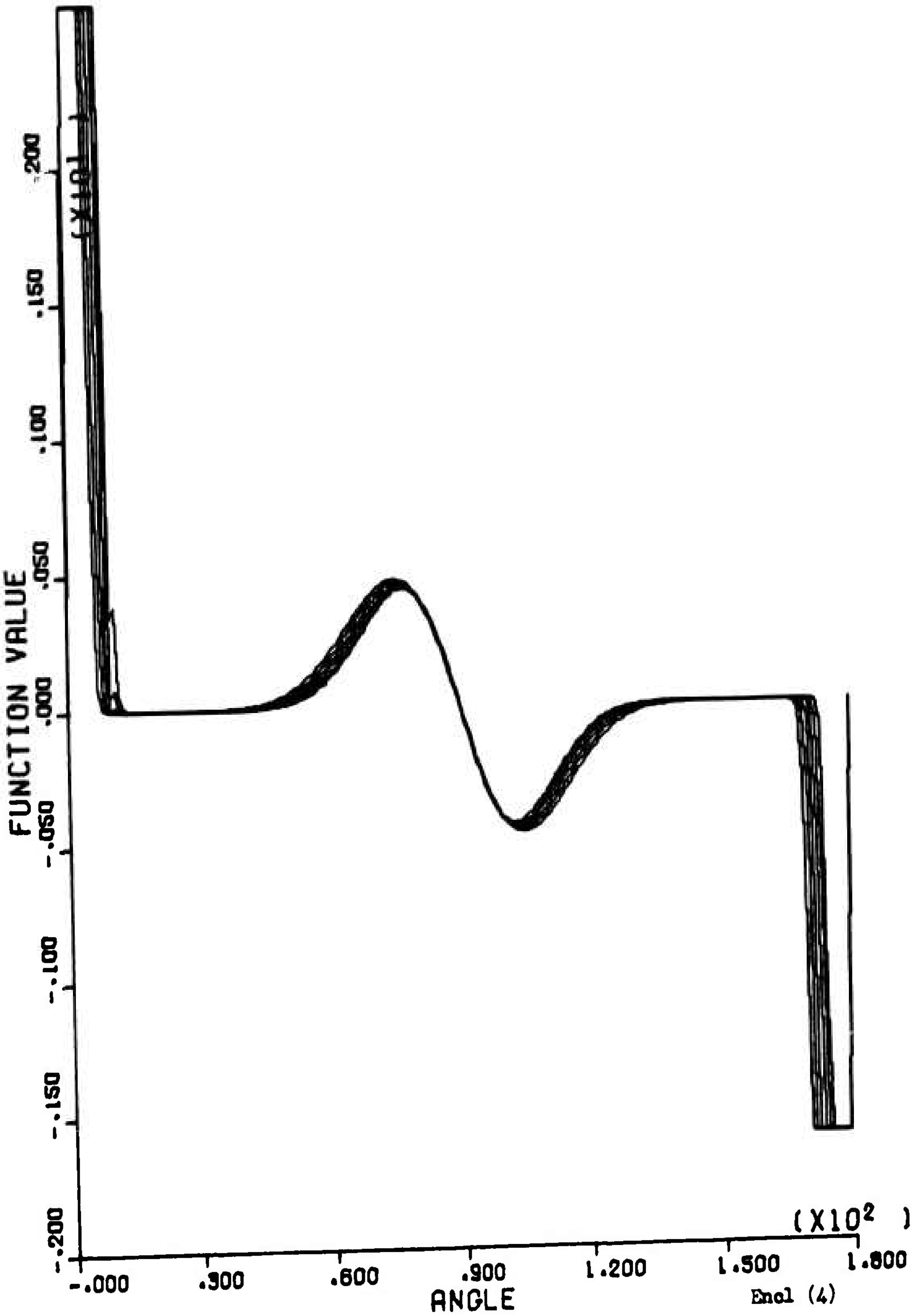
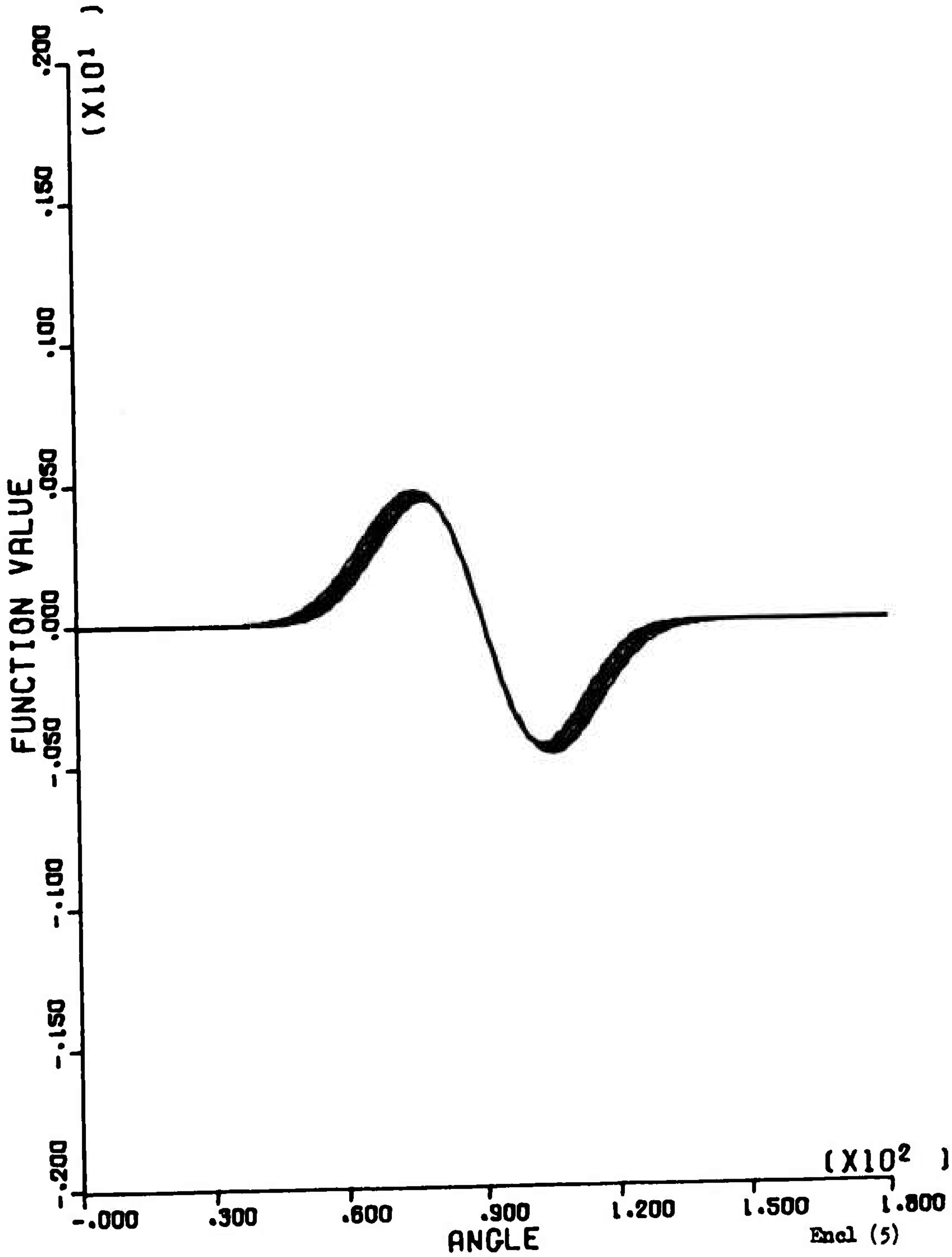


FIG. 5. STABLE PROCEDURE FOR $P_n^{n-1}(\cos\theta)$ FOR n = 15 THROUGH 20



Encl (5)

APPENDIX A

LISTING OF FUNCTION SUBROUTINES

```
      FUNCTION SPNM(N,M,X)
      SPNM=PNM(N,M,X)
      IF(M.EQ.0.OR.M.GT.N) RETURN
      SPNM=SQRT(2.*ANF(N-M)/ANF(N+M))*SPNM
      RETURN
      END

      FUNCTION PNM(N,M,X)
      PNM=0.
      IF(M.GT.N) RETURN
      IF((ABS(1.-ABS(X))).GT.1.E-9) GO TO 20
      IF(M.NE.0) RETURN
      PNM=-1.
      IF(X.GT.0..OR.N.EQ.INT(FLOAT(N)/2.+.1)*2) PNM=1.
      RETURN
   20 CNM=(2.**N)*ANF(N)*ANF(N-M)
      PNM=1.
      IF(M.NE.0) PNM=SQRT((1.-X*X)**M)
      CNM=ANF(2*N)*PNM/CNM
      PNM=1.
      IF(M.NE.N) PNM=X**(N-M)
      IF(N-M.LE.1) GO TO 40
      PRD1=1.
      NT=(N-M)/2
      DO 30 I=1,NT
      AN1=FLOAT(N-M-2*I+2)
      AN2=FLOAT(2*I)
      AN3=FLOAT(2*N-2*I+1)
      PRD1=-PRD1*AN1*(AN1-1)/(AN2*AN3)
      NE=N-M-2*I
      AN1=1.
      IF(NE.GT.0) AN1=X**NE
      PNM=PNM+PRD1*AN1
   30 CONTINUE
   40 PNM=CNM*PNM
      RETURN
      END
```

```
      FUNCTION ANF(N)
      DOUBLE PRECISION AN
      ANF=1.
      AN=1.D0
      IF(N.LT.0) PRINT 01
      IF(N.LT.2) RETURN
      DO 10 I=2,N
   10 AN=AN*DBLE(FLOAT(I))
      ANF=SNGL(AN)
      RETURN
   01 FORMAT(37H1FACTORIAL INTEGER IS LESS THAN ZERO.//)
      END
```